Chapter 3: On Darwin and Mathematics

§3.0 Introduction

‘What is mathematics?’ that is the question we want to expose ourselves to in this chapter. Mathematics is all around us, and it is getting closer and closer upon us. That makes this question all the more persisting. The *design* of the objects that we use, from the shovel till the keyboard, is inconceivable without mathematics; the *structure* of the spaces in which these objects are used, from public spaces till commercial buildings, is ordered mathematically; and the economic *system* in which these objects and these spaces get their values is mathematical to and through.

But these things seem to be *applications* of mathematics, not mathematics itself. We can see shovels and keyboards, parks and buildings and even a lively economy. But where is the *mathematics* in there? Is not real mathematics something theoretical, done by professors in small old rooms at the department of mathematics? Those rooms are full of sheets filled with small symbols, books with difficult titles and computer screens running strange programs. Is mathematics really in there? And if so, how come that this *theoretical* activity transforms our world?[1] How can it become practical? But in what other way then that it is already practical? Is the herdsman, who counts his sheep and when he sees he is missing one starts looking for it, then not a better example of a mathematician?

Hence, we are looking for something that is both practical and theoretical, that is both all around us and not localizable (as we can locate an object or an activity). What can that be?? The question returns, what *is* mathematics?

The structure of this question is metaphysical. It asks, what makes mathematics mathematics; it asks what mathematics qua mathematics is. It asks about the *essence* of mathematics. This essence is supposed to be valid for all mathematical ‘things’, *wherever and whenever* we may encounter them. But we saw in the previous chapter that Darwinism destroyed these metaphysical essences. Thus are mathematical forms an exception or is the question we are asking a dead end right from the start? Even more ironically, the metaphysical question itself

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[1] See Karl Pearson, one of the founders of statistics in the 20th century, in N. Rose (ed.) Mathematical Maxims and Minims, Raleigh NC: Rome Press Inc., 1988 ‘The mathematician, carried along on his flood of symbols, dealing apparently with purely formal truths, may still reach results of endless importance for our description of the physical universe.’

came into existence, because philosophy tried to align itself to mathematics.[2] Thus the question we ask about mathematics qua mathematics springs forth from metaphysics which tried to align itself to mathematics. A clear circle. We first need to understand what mathematics is to understand the metaphysical question about mathematics. Or the other way round: the clearest example of metaphysical essences were the ideal figures of mathematics, the ideal triangle, the ideal circle etc., hence if we ask in a metaphysical way about mathematics we can only get these ideal entities back.

Darwin thus makes it harder and harder for us to think about mathematics, since his theory blocks the obvious question about mathematics, what it *is*? At the same time he opens up the possibility to think about mathematics without being blinded by metaphysical essences. This is empirical, because metaphysics is dead, but mathematics is more alive than ever. Metaphysics simply didn't reach far enough to ask back what mathematics is.

How to ask about mathematics now? First of all we might ask how Darwinism itself thinks of mathematics in more detail. Because Darwinism is a mechanical, i.e. mathematical theory (see chapter 2 on mechanization), this question seems like a form of bootstrapping. Darwinism is a mathematical theory, and using this mathematical theory we ask back about the origins of mathematics. A clear circle too. Should we not just stop here? Does not this circle already show that mathematics should be treated non-Darwinistically? If math is the basis of Darwinism, how can Darwinism ever ask back about it?

Or are metaphysical distinctions still blinding us here? These questions seem to regard mathematics as the atemporal form in which the matter of the Darwinian theory has to fit. But is this scheme of the content of a science versus its mathematical formulation (that is universal and applicable to other sciences as well) not a metaphysical relic in the way we think about science? What if the relation of Darwinism and mathematics is not one of matter and form? A reason to be

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[2] See Oswald Spengler, *Untergang des Abendlandes, Vom Sinn der Zahlen*, 'Until now all philosophies grew in connection with an accompanying mathematics [zugehörigen Mathematik] (p.76).' See also Plato's famous sign above the academy: Let no man ignorant of geometry enter. Or Kant, Kritik der Reinen Vernunft BVII: 'Whether the treatment of such knowledge as lies within the province of reason [i.e. metaphysical knowledge] does or does not follow the secure path of a science, is easily to be determined from the outcome (…). In the earliest times to which the history of human reason extends, mathematics, among that wonderful people, the Greeks, had already entered upon the sure path of science (…). Their success should incline us, at least by way of experiment, to imitate their procedure, so far as the analogy which, as species of rational knowledge, they bear to metaphysics may permit.'

And even after the demise of metaphysics, philosophy still tried to align itself to mathematics. Bertrand Russell: 'To create a good philosophy you should renounce metaphysics but still be a good mathematician.' Or the founder of thermodynamics, Lord Kelvin: 'Mathematics is the only good metaphysics.'

suspicious indeed is that this scheme of *matter and form* is exactly one of the basic metaphysical schemes that Darwinism proofed to be wrong in biology. Why should it still count in math?

We will see that Darwinism doesn't hold mathematics to be a peculiar human activity as many people, metaphysicians and mathematicians alike, think and thought.[3] The process of natural selection is itself a huge *mathematical* algorithm and this algorithm gives rise to sub-algorithms that solve typical problems that can be recognized as problems humans have solved too.

This will become clear when we discuss, shortly hereafter, the natural algorithm of evolution (§3.1). The obvious question afterwards will be to ask whether human mathematics differs from this natural math. And if so, how (§3.2)? When we concentrate on human mathematics more specifically (§3.3) we will have to talk about its actual practice (§3.31) and its history (§3.32). After that we can easily see the status of the so-called philosophies of mathematics (§3.33). Their unsatisfactory character forces us to take a different path. We will start by thinking on the relation of mathematics and number. It will turn out that number is irreducible, despite many attempts to reduce it (§3.41). It will turn out to be neither something of the things, nor something of us. Number is in-between. In science number seems to be this in-between, between the researcher and the world. We will thus next focus on the relation of math and science, especially on the mathematical design of nature (§3.42). The *words* of Descartes and Leibniz can guide us here. The relation of the reflexive cogito and its mathematical 'objects' will make it possible to understand Darwinian mathematics anew. In the final section we will go deeper into the relation of words, of language, and mathematics (§3.5).

§ 3.1 Darwinian Mathematics

Darwin himself was well-aware of the role of mathematics for his theory: 'Every new body of discovery is mathematical in form, because there is no other guidance we can have.'[4] But if we look at his three basic principles, they don't seem to be mathematical at all. In Newton famous

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[3] See P.W. Bridgman, *The Logic of Modern Physics*, New York, 1972, p. 15 'It is the merest truism, evident at once to unsophisticated observation, that mathematics is a human invention.' Gauss in a letter to Bessel of 1830 'We must admit with humility that, while number is *purely a product of our minds*, space has a reality outside our minds, so that we cannot completely prescribe its properties a priori [my italics].' Einstein, Geometry and Experience, p. xx 'How can it be that mathematics, being after all *a product of human thought* independent of experience, is so admirably adapted to the objects of reality [my italics].' Alfred North Whitehead, *Science and the Modern World*, p. xxx 'The science of pure mathematics (...) may claim to be the most original creation of the human spirit.'

[4] In N. Rose (ed.) Mathematical Maxims and Minims, Raleigh NC: Rome Press Inc., 1988. In this quote collection too from Darwin: 'Mathematics seems to endow one with something like a new sense.'

mechanical theory, set forth in the Principia *Mathematica* Philosophia Naturalis, the mathematical side is clear – on almost every page of his book we encounter formula's and geometrical demonstrations; Darwin's *Origin of Species* on the contrary is far more prosaic; it doesn't almost contain any equations or numbers.

The mathematical side of Darwin's theory

However, if we look carefully at the three basic principles of Darwinism: replication, variation and selection, we can see that they are mathematical to and through. Replication is a special form of *multiplication*, in which two 'units' (copies) result out of an original unit. Variation creates small *quantitative* differences (on phenotypic level for instance *stronger* teeth, *faster* legs etc., on genotypic level different base combinations). Selection, finally, has a quantitative aspect as well: the selection is a reduction of the number of replicators in each generation.

Darwin himself gives a quantitative example of the first and the third of these principles. Selection only occurs if not all the animals replicate themselves. Darwin thought that his fellow scientists would not believe this. Hence he took the example of one of the slowest breeding animals, the elephant, and – by assuming that the earth was only 6000 years old and God created just a single couple of elephants back then as the theologians of his day thought – he calculated, estimating a probable minimum rate of increase, that there would 'at the end of the fifth century (...) fifteen million elephants, descended from the first pair.'[5] This is clearly nonsense, thus not all animals replicate themselves; selection exists.

More important is, however, how selection gives rise to all the 'endless form most beautiful and most wonderful,' to quote the last words of the *Origin of Species*, i.e. more important is how selection *shapes* the evolution, gives rise to all the qualitative forms. This selection process is a ratio-nal process, i.e. proceeds according to ratios, tiny differences. Darwin himself gave in the *Origin of Species* the example of a dove keeper who rationally selects according to *tiny differences* which doves are to be crossbred to get even fitter doves. Then Darwin adds that nature itself is just one big dove keeper, nature itself is rational. The Darwinian principle of *selection*, a word normally only applied to humans, to farmers who select their crops, expresses just that: the word 'selection' comes from se-ligo, to interpret. Therefore: when Darwin says that nature selects, he says that nature interprets, that nature thinks! although of course not consciously (which is no problem for Darwinism since it regards pure consciousness, just as every other spiritual entity, as a non-agent).

[5] Darwin, quoted by Daniel C. Dennett, *Darwin's Dangerous Idea* (1995), p. 40.

This non-conscious thinking without a thinker is a form of *calculation*, in which often apples and oranges are compared, not as in economics on the basis of a single currency (for instance 'dollars'), but on the basis of replication value. Thus we could also say, nature *calculates*. Mathematics is not something exclusively human; nature itself is a big mathematician.

Dennett and algorithms

Dennett gives an important elucidation of this non-conscious thinking without a thinker, this form of calculation, by understanding it as an *algorithmic* process.[6] There are a lot of things wrong with Dennett's analysis, but it will also help us to better understand what 'Darwinian mathematics' is. Dennett characterises these algorithmic processes by their a) substrate neutrality, b) underlying mindlessness and c) guaranteed results.

'[Ad a:] The procedure for long division works equally well with pencil or pen, paper or parchment, neon lights or skywriting, using any symbol system you like. The power of the procedure is due to its logical structure, not the causal of the materials used in the instantiation, just so long as those causal powers permit the prescribed steps to be followed exactly. [Ad b:] Although the overall design of the procedure may be brilliant, each constituent step, as well as the transition between steps, is utterly simple. How simple? Simple enough for a dutiful idiot to perform – or for a straightforward mechanical device to perform. (...) [Ad c:] Whatever it is that an algorithm does, it always does it, if it is executed without misstep. An algorithm is a foolproof recipe.'[7]

It may seem strange to understand the natural process, a process in which variation and coincidences play such a mayor role as an algorithmic process, i.e. a process with a guaranteed result – didn't we say in the previous chapter that if the process of evolution started again with a little bit different initial conditions the outcome would have been radically different? And indeed the characterisation of Darwinian evolution as a mathematical, as an algorithmic process, remains vague, if we don't say what *kind of algorithm* evolution is.

To explain the kind of algorithm he thinks of, Dennett asks us to imagine a huge library he calls the library of Mendel. This library is like the library of Babel from a story of Borgess, which contains all possible books (of a certain length), because every possible combination of letters is available as book in this library, for instance a book with 500000 A's, another with 499.999 A's and a B at the end, another with 499.999 A's and one C, another with 499.998 A's a

[6] Dennett, Darwin's Dangerous Idea, Penguin New York, 1995, p. 51 'We can now reformulate his [=Darwin's] fundamental idea as follows: "Life on earth has been generated over billions of years in a single branching tree – the Tree of Life – by one algorithmic process or another." '
[7] Ibid.

B and a C at the end etc. Hence this library also possesses a copy of Moby Dick, many copies of Moby Dick with one fault, even more with 2 faults etc. In contrast to the library of Babel, however, in the library of Mendel the 26 letters (and 10 or so punctuation marks) are replaced by the 4 bases of DNA: A, T, G and C (Adenine, Thymine, Guanine, Cytosine).

This library of Mendel is a '*logical* [design] space (idem, p. 110)' that contains ' "all possible" genomes (p. 112).' Evolution is now the process in which out of all the different 'books', due to simple mechanical operations, only the Hamlets and the Moby Dicks remain. How so? Viable 'books', viable genomes *replicate* themselves so they remain; due to simple mutations *variations* (shifts in the library) occur; most of them give extra errors, and selection kills them, although sometimes these variations are viable, or even better than the original and thus replace the original volumes.[8] In Dennett's own words:

> 'The library of Mendel (or its twin, the library of Babel – they are contained in each other, after all) is as good an approximate model of Universal Design Space as we could ever need to think about. For the last four billion years or so, the Tree of Life has been zigzagging through this vast multidimensional space, branching and blooming with virtually unimaginable fecundity, but nevertheless managing to fill only a vanishingly small portion of the space of the Possible with actual designs. According to Darwin's dangerous idea, all *possible* explorations of Design Space are connected. Not only all your children and your children's children, but all your brainchildren's brainchildren must grow from the common stock of Design elements, genes and memes, that have so far been accumulated and conserved by the inexorable lifting *algorithms* [my italics], the ramps and cranes and crane-atop-cranes of natural selection and its products (p. 143f).'

There are a lot of problems with this view. First of all, not all possible combinations are included in this library because the size of a book or a string of DNA can be longer than the 500.000 letters, base pairs (or every other set of elements). More importantly: all possible 'elements' are not included. RNA has for instance Uracil (U) instead of Thymine as its forth element, so all RNA strings (the RNA viruses) are excluded. Dennett seems to realise this as he admits that the library of Babel doesn't contain books written in different characters such as Russian and Chinese (p. 112). Since the number of characters and elements, is not determinable, the 'a priori' idea of a library of all combinations seems to be a hopeless monstrosity already. But an even bigger problem is the inclusion of the library of Babel into the library of Mendel, the library of design space. You need extra elements for this as well, because both are written in *different* units (DNA

[8] Dennett gives the funny example of a one letter variation in the words of Kain: 'Am I my brothe*l*s keeper (p. 110)?'

bases and letters). But the biggest problem is how to organise this library? What ana-logy can we use to write down its catalogue?

Even if we equate for convenience the library of Babel with all the possible books written or transcribed into English, the question returns, were can we find all the gestures, the artefacts such as tables, spoons, computers, flats etc., that are replicators too in the Universal Darwinian theory? Dennett had better read another story of Borgess about 'a certain Chinese encyclopaedia' with an 'impossible' ordering: 'the animals [in of the lemma of this encyclopaedia] are ordered as a) animals that belong to the emperor, b) mummified, c) domesticated, d) sucking pigs, e) sirens, f) fable creatures, g) dogs that walk about freely etc.'[9] Dennett's library of all replicators can but have the same impossible ordering.

Put differently, even if genes and memes are, according to Dennett, subject to the same algorithmic *logic*, by what *ana-logy* can we a priori determine a system principle that will provide a categorisation for all possible replicators, genes, memes etc.? How can we a priori identify a space of basic elements out of which all possible replicators are built? Universal Darwinism shows that new basic elements of replication can always arise (hence there are none).

At the basis of all these problems lies the idea of Design Space as a *logical* space. Replicators are equated with their *logical* combination of basic units. Some of these combinations are meaningful others not. This seems to work nice for the library of Babel: the fortuitous combination of letters that makes up Hamlet is highly meaningful, the book that contains only A's not. But the library of Babel forgets the background against which 'Hamlet' is meaningful: people speaking English, people living under a certain constitution with a king, people experiencing complex emotions etc. Only because of this background can Hamlet be meaningful, and thus a successful replicator (it wouldn't have been in the times of the Neanderthals). Just the same holds true in the case of DNA. My DNA may consist out of a combination of a billion bases. This combination is not *in itself* meaningful, it is only a fruitful replicator because other replicators (other base combinations) exist at the same time: the base combinations of the bacteria in my intestines, the base combinations of my food etc. The human base combination would be meaningless in the time of the protozoan bacteria two billion years ago when plants didn't exist yet. A DNA string is only meaningful, because it *codes* for a certain phenotype that in a certain *time and space* is thus well designed that, while fighting against the natural elements, its natural enemies and its congeners, manages to replicate itself (and thus these genes). This *time and space*, this 'world' is, however, completely absent from Dennett's a priori, logical library of Mendel.

[9] Quoted in the preface of 'the words and the things' by Foucault.

The emphasis on logic also invades Dennett's description of what constitutes an algorithm. The third characteristic of an algorithm, its infallibility, is the infallibility of *logic*. However, real implemented algorithms – instead of idealised algorithms – can always go wrong.[10] In the first characteristic this emphasis on logic becomes most apparent, as he wrote: 'The power of the procedure [i.e. the algorithm] is due to its logical structure[11], not the causal power of the materials used in the instantiation.' In other words: the algorithms are substrate neutral according to Dennett. This is but a modern formulation of the *metaphysical* distinction of atemporal mathematical and logical truths versus temporal empirical physical facts. But an algorithm – its *success* – is not substrate neutral, let alone independent of 'any symbol system you would like to use'. Try a long division in roman numerals! An algorithm is always designed for certain symbols. Moreover, the speed and accuracy of the result of an algorithm in its actual use is highly determined by its success. Nobody still calculates a multiplication of two numbers of more than 4 numerals by hand: it is too slow and you make too many mistakes. An algorithm is as good as the trust that can be put into its success. Some algorithms for, for instance, the numerical solution of a differential equation are so long, that no human, given the length of his life, could solve it: big computers with big memory banks are needed. These memory banks are part of the substrate of the algorithm, which itself must include rules how and where to store the large numbers of its in-between results.

Hence, the substrate neutrality is a fiction. Of mathematics is not the same as physics, a science of the possible substrates; mathematical proofs are not about atoms, but algorithms are not floating freely, unbound by matter, in a Platonic heaven either. But where are algorithms then? The next section will make this clearer.

We saw that we had to say that nature calculates. Understood in the right way, without the a priori construction of a library of all possible entities (possible 'essences'), the rule based algorithmic process can indeed elucidate this. We will describe three (sub)algorithms that arose in the process of natural selection to show this.

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[10] See Wittgenstein, *PI* 193 'The machine [a rule based device] as a symbol of its way of functioning: (…) the machine seems to have its way of functioning already in itself. What does that mean? Because we know the machine, all else such as the movements it will make, seems to have been determined already. We speak, as if these parts can only move in this way, as if they cannot move differently. How come? Do we thus forget the possibility that they bend, break, melt etc. Yes, in many cases we do not think about this. We use a machine, or the image of a machine, as symbol for its peculiar way of functioning. (…) But we don't speak like this, when the question is to predict the true behavior of a machine. Then we don't forget, generally, the possibility of a deformation of the parts etc. (…) The movement of a machine symbol is differently determined in advance than that of a certain real machine.'

[11] It is important to see that the mathematical algorithm is thus dependent on logic. Logic is thus more fundamental than mathematics? We'll come back to this when we speak of the structure of proofs, of logicism and finally in the last paragraph of this chapter on mathematics and language.

Three examples of Darwinian math

In the huge algorithmic process of natural selection, sub-algorithms evolve. In the DNA of bees for instance a sub-algorithm evolved that made them build the cells of their hive in a hexagonal shape. Not all bees in the past did this. There are bee hive fossils with square cells. The optimal way to cover the largest area with small cells under the condition of the least amount of work, i.e. with the smallest number of walls is to use the hexagonal structure. This is a very hard thing to proof, but it is clear how this hexagonal structure could have evolved. That colony of bees whose queen had a certain mutation that caused her worker ants to produce (more or less) hexagonal instead of square cells had a systematic advantage – they needed less material to build the cells – compared to other bees, hence they outcompete them and the solution to the optimal area filling problem was carried along in the DNA of the bee queens.

Another example: the way ants find the shortest path to a food source. Ants walk in lines after each other. To achieve this all of them have an organ that secretes a constant amount of a certain liquid with a strong smell. If an ant smells this, it will follow its trace.[12] Hence, if a first ant secretes this liquid, a second ant will follow him secreting the same liquid etc. If ants smell a certain food source they will walk in a line towards it. But the remarkable thing is that they even succeed in doing this, if it lies behind an unknown object.

Due to their small size ants miss the overview over their territory to immediately see the shortest route, clockwise or anticlockwise around this object. They are, however, programmed with 2 additional rules: 1) follow the trace that smells most, 2) if the smell of two traces is almost equal do the opposite of the ant in front of you. Hence, a clockwise and an anticlockwise path around the object towards the food source are formed.[13] The path that is shortest will soon have the strongest smell, because the smell of this path gets updated more often (because the returning ants come back sooner and add to the smell of the trace). Thus after a little time all the ants at the crossroad choose this shortest path!

It is clear how this algorithm could have evolved. Rule 1 evolved to make sure that if ants felt a little doubt which smell to follow, they would stay together. Rule 2 evolved accidentally as a possible solution of what to do in ambiguous situations. Combined they make up a very effective 'find the shortest path' algorithm which is hard coded in the DNA of ants. This

[12] This smell has to be above a certain threshold value. Otherwise the ants would always run in circles behind each other. If they smell a food source and no trace of a smell above the threshold value, they will head on their own towards the food source.

[13] The first ant that reaches the object chooses for instance the clockwise direction. The smell of his trace is still weak, *not above the threshold*. A second ant smells the food source too and might choose the counter clockwise direction. This is how the process gets running.

algorithm is so effective that human mathematicians use it too and call it the ant algorithm; the biannual world conference on the theory of algorithms even calls itself Algorithmic Number Theory Symposium (ANTS).

Not only problems in geometry, also problems in pure arithmetic are solved in nature. The life cycle of certain cicadas is a good example of this.[14] These small creatures show up every 13 or every 17 years, and thus *not* every 12, 14, 15 or 16 years. These creatures use the prime character of 13 and 17. The reason for this is an evolutionary one. It is assumed these creatures were once in an arms race with a now extinct predator, and by showing up only once every 13 or 17 years they make it very hard for this predator to synchronise itself with them. If the predator looked for these creatures every 3, 4 or 6 years it would have success with the 12 year cycle in respectively 25%, 33% and 50% of the cases; if it looked for these creatures every 5 years it would have success with the 15 year cycle in 33% of the cases, etc. But in the case of the 13 year cycle the predator can only have (lasting) success if it looks every 13 years.

It is clear how this characteristic evolved. Those creatures that due to a certain mutation showed up every 13, instead of for instance every 12 years, had a systematic advantage to their fellow creatures. Hence, there was a selection pressure on having a cycle of a prime number of years, which became implemented in the genome of these creatures. Here, nature uses the oldest human algorithm ever: the so-called Sieve of Erastosthenes – a crude algorithm to find prime numbers. Write down all the natural numbers, start with the lowest: 2[15], cross out all multiples of this number (4, 6, 8, 10 etc.) and go to the next number 3, cross out all multiples of this number too (6, 9, 12 etc.) and go to the next number 5, cross out all multiples (10, 15, 20 etc.) etc.; the numbers that are not crossed out are primes. Nature, in letting those cicadas get killed whose life cycle was a multiple of a low prime number, literally crosses DNA strings out that coded for such life cycles.

The solutions to all the above problems are written down in the DNA. Due to the natural selection on multiple generations other possible solutions - DNA strings - are ruled out (12 year cycles, squares as optimal area filling etc.). But DNA itself can also be used to compute things in so called DNA computers. These computers use DNA strings as storage devices because the strong A-T and G-C connections can be used as binary bits. There are still many problems with these computers (humans still have to prepare the input strings etc.), but the main advantage is that DNA strings can store an incredible amount of information in a very small volume: cells are

[14] See Dawkins, *the extended phenotype*, p. xxx
[15] Erastosthenes actually started with 3, but that need not worry us. See Classics of Mathematics, Ronald Calingher (ed.), Prentice Hall London 1995, p. 115.

very small, cell nuclei are even smaller, yet still every human cell nucleus stores – when stretched out – a DNA string of 2 meters long.[16] This is possible due to the incredible folding mechanism of the DNA-helix, which itself of course resulted from the very efficient algorithmic search of good replicator molecules earlier in the natural evolution. To calculate how a simple relatively short enzyme folds itself stereometrically a modern PC has to calculate for many, many hours. The folding of a DNA molecule itself which codes for many, many proteins (enzymes) is as yet absolutely impossible. Here is thus a problem natural evolution can solve, but human computers can't.

Natural evolution is thus the greatest mathematician ever, using DNA instead of numbers or figures as its *signs*. The math gene – a (collection of) genes – Keith Devlin posited human must have to do math – is thus a bit of a strange expression. Every gene is already mathematical to and through. On the other hand, it is indeed the question whether humans do math in the same way as natural selection. It seems to me a truism that there are genes involved in men's mathematical abilities, just as there are genes involved in every other human activity, to walk, to eat, to speak etc. But this truism doesn't learn us anything about the way humans do math.

§3.2 Darwinian math and human math

A lot of differences spring to mind:

First difference: proof, certainty and intuition

1) 'Human mathematics proofs something for all possible cases beyond doubt, natural selection doesn't proof anything'.

This sentence contains three claims: a) human mathematics proofs something, whereas natural selection does not, b) these proofs are for all possible cases, c) these proofs are absolutely certain.

However, the first claim a) is simply wrong: the algorithms of natural selection are *proofed*, are tested *all the time*. If they don't work any more, they are left behind. The algorithms proof themselves, they are still there. 'But the proofing of a human mathematician is not just

[16] See the *McGraw Hill Encyclopedia of Science and Technology*. New York: McGraw Hill, 1997, article human DNA. This means that the total length of DNA present in one adult human is:
= (length of 1 base pair) x (number of base pair per cell) x (number of cells in the body)
= $(0.34 \times 10^{-9}$ m$) \times (6 \times 10^{9}) \times (10^{13})$
= 2.0×10^{13} meters
That is the equivalent to almost 70 trips from the earth to the sun and back.

testing *here and now*, it is done for *all* times, something cannot possibly turn out false after being proofed.'

The last two claims tell us what counts as proofing; for the mathematician something *is* a proof if b) its *truth* is demonstrated for *all* possible cases and c) beyond any *doubt*.

b) Darwinism equates truth in mathematics with that what works; the metaphysician with some special insight, an intuition that is supposed to grasp all by itself the clear essence of a mathematical structure. Wittgenstein in his *Philosophical Investigations* destroys this metaphysical appeal to intuition (PI 185). One can never foresee *all* possible cases in math, the essence does not contain them. Wittgenstein asks us to imagine a student learning the row 2, 4, 6, 8, 10, 12 etc. After he seems to have understood this row, the student is asked to apply the rule he learnt to the number 1000. However, instead of writing down 1002, 1004, 1008 etc. as we would he writes down 1004, 1008, 1012 – as if he changed the rule 'add two' into 'add 4' when the numbers contain 4 or more digits.[17] 'We say to him, "Look what you have done!" But he does not understand us. "You had to add two [the 'essence' of the row]; look, how you have started the row!" – He answers: "Yes! Is something wrong? I thought that's the way I should do it." – Or assume he said pointing at the row: "I did continue the row in the same way" – It wouldn't help us at all to say "But don't you see….?" – and to repeat the old explanations and examples' (PI 185). No intuition, no essence exists that can guarantee somebody understands how to continue the row. And even if such an essence existed, I wouldn't be able to communicate it to someone else, and it may therefore just as well not exist (the function of an intuition in our language games would be the same as that of the famous beetle in a box that only I can see.) Even the formula of the row (n+2) does not 'contain' in advance all the cases to which it applies, because a formula, just as the row, is *its use* (PI 189).

Wittgenstein's example may be a bit dull, but in real mathematics special cases always destroy rules that turned out to be false generalisations, i.e. rules that turn out not to be clear for all possible cases. An example for the mathematician on the extra conditions for integration: first mathematicians thought that 'all' functions could be integrated; then that all continuous function could be integrated; then that all *'well-behaved'* continuous functions could be integrated, for

[17] Or to 'if the number is above 100'; or 'if I have done the experiment already three times before', or 'if the second digit is a zero' etc. It is impossible to be certain absolutely what rule somebody is following, because every instance of use, tests the (supposed) rule again. And here we are more tolerant in assuming somebody makes a mistake with children than with grown-ups.

which first Riemann, then Riemann-Stieltjes, then Lebesque and finally (up until now) Henstock-Kurzweil formulated more and more stringent conditions.[18]

'But of course, the math may become more refined, but with for all possible cases I meant that if something is sound in mathematics, it will remain that way forever, the evolving arithmetical processes lack this quality, they lack the absolute certainty mathematics is famous for.'

c) This understanding of the word 'certain' as absolutely certain is metaphysical too. The certainty of math is not to be grounded in the success of its use, but in something higher, which explains both this certainty and its success. This 'something higher' is always the *clearness* of an *intuition*. In math this clearness is twofold. First, the clearness of the *axioms*, the first principles. Second, the only moves that are allowed to a mathematician are steps that follow *logically* from the combination of these axioms. Such steps are clear and thus called *proofed*. Clear axioms and clear logical steps are the core of the standard model of deductive reasoning. Its classical paradigm is Euclid's *Elements*.

Putting aside questions whether Euclid really lives up to this standard of this model and whether Greek *axioms* are really the same as our first *principles*[19], we can see two moments in this model at which uncertainty can slip into the proofs: a) because the axioms turn out not to be clear at all, b) because there are so many steps that it becomes hard to check whether the argument is still logical. This is not just the pettiness of a philosopher, trying to shake indubitable knowledge. In mathematical texts one can often find phrases such as 'a more rigorous proof can be found' or even 'a more natural proof' etc.[20] These phrases reveal that mathematicians themselves acknowledge these problems; a proof is sometimes not a real proof.

The mathematician and philosopher Descartes already saw this second problem, and in his *Rules to direct our Mind* he adds a long rule to try to get around it: he suggests that we should review a proof many, many times, once slowly, then again quick, until we have a complete

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[18] Mathematics is thus not just dealing with higher and higher abstractions. See Paul Halmos, *I Want to be a Mathematician*, Washington: MAA Spectrum, 1985, p. 23 'The source of all great mathematics is the special case, the concrete example. It is frequent in mathematics that every instance of a concept of seemingly great generality is in essence the same as a small and concrete special case.'

[19] On flaws and lacks of rigor in Euclid, see *Classics of Mathematics*, ed. By Ronald Calinger, Prentice Hall 1982, p. 79. On axioms as different from principles, see Heidegger, *Frage nach dem Ding*, p. 71. Principles are *grasped* by the subject, axioms are being paid tribute to by men. See on the difference of Greek and modern math the paragraph on human mathematics.

[20] First quote from *Classics of Mathematics*, ed. By Ronald Calinger, Prentice Hall 1982, p. 178. Second quote from one of the math lectures I attended. The word natural in this quote is ambiguous. Is the proof more natural because it does more justice to the mathematical structures or because it is easier for us to grasp.

overview of the whole proof and each of its subsequent steps.[21] This problem of having overview returns nowadays in proofs done by computers, because they contain too many steps for man to check. Although the results of the computer proofs are 'proofed' again and again by their success in practice, many mathematicians *say* they still want to have a subjective overview of a problem.[22] The first proof done by a computer, the 4-colour problem, thus caused a lot of stir in the mathematical world. However, nowadays a lot of proofs are partly or completely done by a computer.[23] The current mathematical practice thus *shows* we *trust* a computer more than ourselves.

The first problem of the clearness of the axioms is one in which computers are of no help. How to find true axioms? Or – if one believes that axioms are just tools to open up a certain mathematical field that we are free to choose, only some give more interesting deductions than others – how to secure that the axioms themselves are clear? (And does not this demand of clarity restrict our 'free' choice of axioms in the first place?)

This demand of clear axioms, clear principles, let to the foundational crisis in mathematics at the beginning of the twentieth century. The best attempt to solve this crisis is Bertrand Russell's and Alfred North Whitehead's co-authored *Principia Mathematica*, in three thick volumes. Fortunately, we do not have to go into the details of this unreadable book that proves 1 +1 = 2 at the end of volume 1, because one of the authors himself acknowledges at the end of his life the failure of his project:

'I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers expected me to accept, were full of fallacies, and that, if certainty were

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[21] See Descartes, *Rules for the Direction of the Mind*, rule 11 and its subsequent elucidation: 'If, after gaining intuitive knowledge of several simple propositions, we are to draw some further inference from them, it is useful for us to run through them in a continuous and uninterrupted movement of thought, to reflect on their interrelations and to form, sofar as we can, distinct conceptions of several at once. For this adds much to the certainty of our knowledge, and it greatly increases the scope of our mind.'

[22] See John Horgan, *Scientific American* 269:4 (October 1993) 92-103. 'It would be very discouraging if somewhere down the line you could ask a computer if the Riemann hypothesis is correct and it said, "Yes, it is true, but you won't be able to understand the proof." '

[23] The four colour problem is the hypothesis that you need only 4 colours to draw a map of a world in such a way that all the countries on it – touching each other in whatever possible way – never border a country that is coloured identically. See the later chapters of the popular scientific work 'Four colors suffice' for the criticism the proof received, because a computer was used, since the number of possible ways in which countries could border each other was to big to check one by one by hand.

The old 4-colour press printing is based on the 4 colour hypothesis, just as the 4 color CGA computer screens: 4 colors suffice to create images that cannot cause confusion. Economically speaking, printers could drop a fifth color, four suffice! But if they wanted to cut costs even more by dropping the fourth color, their images would become bad (in practice, printers of course used 3 colors, because they used the white of the background as the fourth color).

indeed discoverable in mathematics, it would be in a new field of mathematics, with more solid foundations than those that had hitherto been thought secure. But as the work proceeded, I was continually reminded of the fable about the elephant and the tortoise. Having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of very arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable.'[24]

Hence is our situation that of a 'mathematics without foundations' as the title of a famous article by Hilary Putnam suggests? Or is the word 'foundation' here still motivated by the old understanding of certainty as the clearness of an intuition? The failed outcome of the foundationalist project learns that there are no certain intuitive foundations to math. But does not are daily praxis show that we do not *need* this knowledge, this intuition? We keep using numbers; if they weren't secure we would stop using them. Our praxis shows that when we use numbers, we are on safe *ground*, numbers are the 'foundations'.[25]

Is this all we can say? Is this the final answer to the question at the beginning of this chapter why mathematics is all around us? Because it gives *us* safety? But didn't we say *nature* uses numbers too?

Second difference: non-natural geometries, number systems etc.

2) 'Human mathematics is "not natural"; its problems can be about things that are not in nature such as non-Euclidean geometries and imaginary numbers etc., whereas natural selection's math is always bound by Euclidean space and natural numbers.'

This argument has two sides: a) human mathematics is not 'natural', b) Darwinian algorithms are natural. But human mathematics is a human activity, humans are natural organisms and so are their activities, so how can human mathematics be unnatural??? 'Well maybe, because man's mathematical *structures* cannot be found in nature, structures like the non-Euclidean geometries and imaginary numbers.' But is then some part of human mathematics, the part about Euclidean geometry and rational numbers, natural, whereas another part isn't??? Is Greek mathematics which was limited to Euclidean geometry and rational numbers[26] natural,

[24] Bertrand Russell, *Portraits from Memory*, p. xxx.

[25] 'Defendit numerus: There is safety in numbers.' Anonymous, in J. R. Newman (ed.) *The World of Mathematics*, New York: Simon and Schuster, 1956, p. 1452.

[26] The Greeks knew irrational numbers such as the square root of 2, but their existence came to them as an unwelcome surprise. The Pythagoreans who uncovered the existence of irrational numbers tried to hide this truth (remember the well known story of the first Pythagorean who made this knowledge public and

whereas our math, from the 16th century onward with its infinitesimals, imaginary numbers and other geometries, unnatural???

And the Darwinian algorithms would be more like Greek math??? That doesn't make sense and is clearly untrue. We saw that Darwinian algorithms can solve problems that are 'unnatural' too – if we want to use this word at all. The problem of the folding of the DNA molecule is a multi-multi-dimensional problem, that has to be modelled in a many-many dimensional 'hyperspace' (many, many free variables, every variable is a 'dimension'). This hyperspace is not the Euclidean 3-dimensional space, but Darwinian algorithms now how to handle it.[27] Another example: the shape and size of the cochlea, the snail's shell. This problem has to be modelled by 'unnatural' infinitesimals, but Darwinian algorithms now how to solve it. On the other hand, the 'unnatural human math' of imaginary numbers can use very simple

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drowned in suspicious circumstances). In book 6 till 8 of Euclid's Elements on number theory, generally assumed to be Eudoxus's theory, the Greeks found a better way to deal with these irrational numbers, by making number theory a branch of geometry. Numbers were identified with the length of a line. One couldn't write down an irrational number as a fraction, but it can be displayed as the length of a line.

[27] One might object that we humans have to model this problem by an 'unnatural' hyperspace, but that Darwinian algorithms solve these problems differently, in a natural way. But these words remain shallow if one cannot point out what this natural algorithm is. Einstein's famous remark: 'God [i.e. nature] does not care about our mathematical difficulties. He integrates empirically,' is completely unintelligible to me.

‘natural Euclidean’ pictures, does that make it more natural?[28] And finally is Euclid indeed as natural as he is thought to be?[29]

In the end all these questions result from the thought that mathematics deals with *things*. Darwinian algorithms are natural, hence deal with *natural things*. Natural things are 3-dimensional objects, thus Darwinian algorithms themselves can only be *about* 3-dimensional object. But mathematics deals with *signs* and the *rules* of their manipulations. All signs have a spatiotemporal side (their index character); they indeed exist within Euclidean space, but not *as sign*. The printed number ‘45’ here consists out of atoms in Euclidean space. But not as *signs*. The number 45 has two parts, the 4 and the 5. But the first number is not a four, it is forty, whereas the second number is indeed a five. This is called the *place-value system*; the *position* of a number in a bigger number is semantically significant. In this way the combination of the four and the five finds its place (its *identity*) in the Non-Euclidean *number space*, in between 44 and 46 (45 is not-44 and not-46 etc, this is the *iconic* character of the number sign). Hence even numbers, are ‘unnatural’ math?

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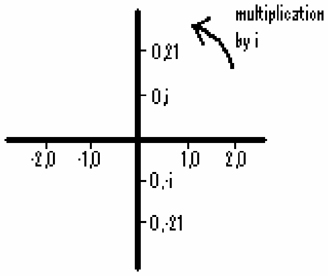
[28] The equation $x^2 = 1$ (a number times itself) has one positive solution x = 1. At high school you learn, however, that x = -1 is also a solution, if you allow negative solutions too. The equation $x^4 = 1$ (a number four times itself) has the same 2 solutions x = 1 and x = -1. If, however, you also allow solutions in the imaginary numbers, the equation has 4 solutions, namely x = 1, x = -1, x = i and x = -i. Where i is defined as i * i = -1 (hence i^4 = i * i * i * i = -1 * -1 = 1). These four solutions can be visualised in a very simple Euclidean picture of two axes. Normally, one of the axes is the x-axis, the other the y-axis. Now the horizontal axis is the non-imaginary number axis, the vertical axis the imaginary number axis. The solutions of $x^4 = 1$ can be displayed by the points (1, 0), (-1, 0), (0, i), (0, -i).



Imaginary number doesn’t mean non-existing number, just as irrational number doesn’t mean stupid number, since a (circular) electrical current, which has to be modelled by imaginary numbers, can kill you right on the spot! (Although the origins of the names may indeed be the fact that the people who first thought of them, didn’t know where to ‘put’ these unfamiliar numbers in their universe, mistakenly understanding mathematics as about objects, instead of dealing with signs. See for example these words of Leibniz, Werke Bd. 5.2, WBG Darmstadt 1997, p. xxx: ‘The imaginary number is a fine and wonderful recourse of the divine spirit, almost an amphibian between being and not being’).

[29] See Spengler, *Untergang des Abendlandes*, p. 91 ‘In spite of the lay man’s judgement of the immediate mathematical evidence of intuition, as we can find in the work of Schopenhauer, Euclidean geometry (…) corresponds only within very narrow borders (“on paper”) approximately with the intuition. What the case for big distances is, shows the simple fact that for our eye parallel lines touch each other at the horizon.’

‘But the Darwinian algorithm gives *imperfect* solutions, the shell of the snail is not a *perfect* spiral, because it contains faults and is made up out of very small atoms with are not true infinitesimals; human mathematics deals with these perfect shells and real infinitesimals’. No again you confuse signs with things. The phenotypic (thing-like shell) is ‘imperfect’, but the DNA(-sign) that codes for it is ‘perfect’ – if you want to use the words ‘perfect’ and ‘imperfect’ at all. This is what we meant at the beginning of this chapter that mathematics is all around us, yet not to be found in keyboards, shovels, public buildings etc. *itself*. The shell is not *itself* a mathematical object, it is mathematical because an algorithm coded for it. But how can algorithms, signs have anything to do with things? And aren’t signs themselves things? We come back to this question when we speak about ‘human mathematics and its application in science’.

Third difference: effectiveness of algorithms

3) ‘Human algorithms work differently, because they can be proofed to be the right procedure, they work *directly* towards the result, natural selection doesn’t.’

There is a subdepartment of mathematics that concentrates on algorithmic calculations, called numerical mathematics. This subdepartment proofs algorithms that can quickly calculate many decimal places for for instance π. One of the main characteristics of these algorithms is that in *each* iterative step they are run, one finds more and more decimal places and hence a better approximation. The algorithms are thus goal directed, but the Darwinian evolution is not goal directed. Look for instance at the evolution of the cells of bee hives: in the first generation all bee hives have square cell walls; in the second generation due to a mutation one has rectangular cell walls; in the third all have square cells again, because rectangular cells are less effective; in the fourth again due to a mutation one bee hive has triangular cells; in the fifth again all have square cells etc., until *accidentally* a bee hive with hexagonal cells arises, which has systematic better survival perspectives. ‘And this is because human math *proofs* theorems, but Darwinian evolution does not.’ Darwinian evolution doesn’t thus work towards the hexagonal shape, whereas the human algorithms seem to be goal directed.

However, the case of the ants shows a different story: the shortest path finding algorithm is always successful. It *proofed* once to be successful and *remained* that way. Of course the way towards the implementation of this algorithm, just as the way to the hexagonal shape of the cells of the bee hive, took a lot of time and – looking with the benefit of hindsight at their evolution – a lot of dead end streets were taken (i.e. unsuccessful mutations that died an early dead). However, in this evolutionary mathematics *does not differ from human mathematics*: many, many mathematicians tried to solve Fermat’s theorem for almost 4 centuries, often claiming they had found the solution – sometimes indeed finding partial solutions – until Andrew Wiles, after 7 hard years found the solution in 1993. Which contained a mistake in the proof... but a year later, in 1994, Wiles did finally find the solution that is generally believed to be correct.

'And Wiles proof is an *impossibility* proof: there are no natural numbers 'n' except 2 for which the equation $x^n + y^n = z^n$ is true for any natural numbers x, y and z. This proof doesn't give an algorithm, but still teaches us something about this equation. And there are many examples of impossibility proofs. In the nineteenth century it was shown that the trisection of the angle, one of the three traditional problems of Greek geometry, was *unsolvable* with the use of a ruler and a compass. And these proofs are not a characteristic of modern math, the Greeks already gave an impossibility proof: it is impossible to find the ratio to of the basis of a triangle with to equal sides to its hypotenuse (i.e. to write the square root of 2 as a fraction of 2 integers).[30] All these impossibility proofs are more *general* and demand an *overview* that nature itself does not have when it calculates. We can proof that no regular 7-edge shaped figure (heptagon) can fill an area without empty space in-between, nature does not; hence we can since this proof eliminate any future attempt to find such an area filling. Nature, however, does not eliminate *in advance* mutations that try to achieve a heptagonal area filling. Nature only knows, so to speak, *positive* mathematics, humans also *negative* mathematics.'

Fourth difference: the character of the algorithms

4) 'Hence not all human math is algorithmic, but all the math of natural selection is.'

We said that the whole process of evolution can be seen as a huge algorithmic process. One thing should, however, not be forgotten. The accidental variation that is an integral part of this process is itself not algorithmic, but chaotic. It is this accidental variation that gives rise to solutions: *due to variation a way out of a problem is found.* The problem of efficiently locating food behind an object was solved by the accidental evolution of the rules ants follow. Hence, the *origin* of the solutions to problems is arithmetic and non-arithmetic: arithmetic insofar as variation is a part of the process of natural evolution, not algorithmic insofar variation itself is non-algorithmic.

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[30] The famous proof can be found in Euclid x.117. It is believed that the Pythagoreans were already aware of the irrationality of $\sqrt{2}$. These impossibility proofs always run the same: they assume that the problem *can* be solved, and then show that this leads to a contradiction, hence the assumption is false and thus its opposite true (the problem *cannot* be solved, in this way).

Aristotle already mentions this proof procedure; he calls it a proof *per impossible* and discusses in the context of the proof of the irrationality of $\sqrt{2}$. (See *Analytica Priora* I.23). We come back to the negativity humans are exposed to when we discuss human math. Intuitionists reject impossibility proofs like these.

In human math the discovery of algorithms is itself not algorithmic too. There have been dreams of an all purpose algorithm to solve mathematical problems all along, from the Lullian Art to Turing's more recent decidability function, that determines whether a certain function can be calculated 'in a finite number of steps based upon a finite number of hypotheses.'[31] But all these dreams have remained dreams. The Lullian art is a fraud and Turing's decidability function is itself not decidable, is itself not calculable.

If we look at the way in which mathematical problems are solved by humans, we see that problems can stand for many, many years, even centuries, until they are demonstrated or is demonstrated that these problems are not demonstrable in this way. We normally don't see the mathematician at work, puzzled, not knowing how to solve a problem, because we only see complete proofs published (albeit sometimes wrong after all). We don't see him experimenting[32], we don't see him guessing.[33] We don't see the mathematician in his aporia, not knowing what to do. And we don't see how suddenly, he finds a way out of his problem, without knowing how he got out. The words of the greatest mathematician ever, speak for themselves:

'Finally, two days ago, I succeeded - not on account of my hard efforts, but by the grace of the Lord. Like a sudden flash of lightning, the riddle was solved. I am unable to say what was the conducting thread that

[31] Hilbert's famous formulation of what counts as a 'mathematical solution' in his lecture delivered before the international congress of mathematicians at Paris in 1900. 'It remains to discuss briefly what general requirements may be justly laid down for the solution of a mathematical problem. I should first say first of all, this: that it shall be possible to establish the correctness of the solution by means of a finite number of steps based upon a finite number of hypotheses which are implied in the statement of the problem and which must always be exactly formulated. This requirement of logical deduction by means of a finite number of processes is simply the requirement of rigor in reasoning. Indeed the requirement of rigor, which has become proverbial in mathematics, corresponds to a universal philosophical necessity of out understanding (Bulletin of the American Mathematical Society, vol., 8 (1902), p. 445).'

[32] Roger Penrose, *The Emperor's New Mind*, Oxford UP, 1999 (1989), p. 124 'The computer is being used in essentially the same way that the *experimental* physicist uses a piece of *experimental* apparatus to explore the structure of the physical world [my emphasis].'

[33] Compare W.G. Anglin*, Mathematics and History, Mathematical Intelligencer*, v. 4, no. 4: 'Mathematics is not a careful march down a well-cleared highway, but a journey into a strange wilderness, where the explorers often get lost. Rigour should be a signal to the historian that the maps have been made, and the real explorers have gone elsewhere.' Paul Hamos, *I Want to be a Mathematician*, Washington: MAA Spectrum, 1985, p. 27 **'**Mathematics is not a deductive science – that's a cliché. When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork.' Richard Feynmann, Nobel Lecture 1966 'We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover up all the tracks, to not worry about the blind alleys or describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work.' See also Niels Abel about Gauss's writing style: 'He is like the fox, who effaces his tracks in the sand with his tail.' In G. F. Simmons, *Calculus Gems*, New York: McGraw Hill, Inc., 1992, p. 177.

connected what I previously knew with what made my success possible.'[34]

But after the discovery, the original feeling of aporia[35] is lost completely. When the solution is seen, it is impossible not to see it anymore. **'**A thing is obvious mathematically, after you see it.'[36] So obvious, that you cannot imagine you didn't see it before; it looks as if the solution lay there already all the time, clear and visible to everyone.[37]

And once seen the solution can immediately turn into a new technique: 'This new integral of Lebesque is proving itself a wonderful tool. I might compare it with a modern Krupp gun, so easily does it penetrate barriers which were impregnable.'[38] And this technique, just as the Krupp gun, transforms the world. Archimedes found the volume of a sphere to be $^4/_3$ times the radius of the sphere the times the area of the circle (hence in total $^4/_3\pi r^3$).[39] Without this knowledge all sphere like objects around us cannot be constructed, or at least not that easily. Cardano found the general solution to the polynomials of the second degree, i.e. equations of the form $ax^2 + bx + c = 0$, a solution better known as the square root formula. Without this formula whole branches of physics (mechanics, electricity etc.) would be impossible and thus lots of technical devices.

Thus the discovery of new math is, in contrast to its use, not straight forwardly algorithmic. But does that mean *non*-algorithmic? We saw that the process of natural solution was not a straight forward process towards higher forms of life (as Spencer thought), but a process with a lot of blind alleys and alternative routes. But due to the force of selection that works upon the many variations, in this process still design could arise, in which even smart solutions such as the prime number cycles could result, which once found remain permanent 'obvious' solutions

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34 Gauss in H. Eves, *Mathematical Circles Squared*, Boston: Prindle, Weber and Schmidt, 1972, p. xxx. .See also Descartes, Discourse on method, p. xxx 'If I found any new truths in the sciences, I can say that they follow from, or depend on, five or six principal problems which I succeeded in solving and which I regard as so many battles where the *fortunes* of war were on my side [*my italics*].'

35 Of Euclid a small book with problems survives that the student studying geometry could use to practice. The title of this book is *Poremata* (*Porisms*): problems you need to find your way out of and of which you can be certain that there is such a way, because otherwise your teacher wouldn't give them to you.

36 R.D. Carmicheal (the discoverer of the so-called Carmichael prime numbers), In N. Rose (ed.) *Mathematical Maxims and Minims*, Raleigh NC: Rome Press Inc., 1988.

37 Robert J. Kleinhenz, The Mathematical Quotation Server (http://math.furman.edu/mqs.html) 'When asked what it was like to set about proving something, the mathematician likened proving a theorem to seeing the peak of a mountain and trying to climb to the top. One establishes a base camp and begins scaling the mountain's sheer face, encountering obstacles at every turn, often retracing one's steps and struggling every foot of the journey. Finally when the top is reached, one stands examining the peak, taking in the view of the surrounding countryside and then noting the automobile road up the other side!'

38 See also Descartes, *Discours on Method*, p. xxx 'Each problem that I solved became a *rule* which served afterwards to solve other problems [*my italics*].'

39 See the works of Archimedes, ed. By Thomas Heath, *On the Sphere and the Cylinder*, Dover 1953, p.1 'Now these properties [the measures of the volumes etc.] were all along naturally inherent in the figures referred to, but remained unknown to those who were before my time engaged in the study of geometry.'

that transform the balance of power in the desert (the predator outsmarted). This process of natural evolution could be understood as an algorithmic process. Is the development of human math with its experimenting, its blind alleys and its many competing strategies not just such an algorithmic process?

Apart from the question of the *origins* of mathematical solutions, we can ask whether all math itself is algorithmic in character. It has to be noted that algorithms can be *really diverse*: there exist algorithms to find prime numbers, algorithms to perform a long division etc., but also algorithms to find the centre of gravity of a triangle (by drawing bisecting lines to the opposite sides of all three angles), or to construct a plane parallel to another one. Although algorithms are in a stricter sense often identified with *numerical* manipulations, and the two mentioned geometrical algorithms can be solved in algebra too.[40]

Turing made most steps towards an algorithmic understanding of math. He first came up with the concept of a so-called Turing machine, a device consisting out of a set of instructions plus an infinite (or very long) tape, the memory of the devices. Instructions are orders to print, or delete a 0 or a 1 on a cell of the tape or orders to put the tape one position to the left or the right. Turing showed how many computable functions like addition, multiplication, determination of a square root etc. could be emulated by his device and even showed that there exists a universal Turing device that can emulate any particular (combination of) Turing device(s). This looks like a universal algorithmic device like our modern computers, and indeed modern computers wouldn't be possible without Turing's pioneering work. It is, however, a different question whether all *computable* functions are computable by Turing computers. The Alonzo-Church thesis (hypothesis) states that this is the case, but has not been proven.

A possibly even more difficult question is whether all math is about *computable* functions at all. Is all math calculation techniques? We saw that human math can give explicit proofs that something *cannot* be calculated. It not this a clear signal that human math is different from natural math, that not all human math is algorithmic? Or does this only stress that mathematics is about calculation. These negative proofs show that something can't be done, that something is *incalculable*, hence accept that math should be about calculations? Many mathematicians think Kurt Gödel's incompleteness theorem proves that there is meaningful math

[40] The problem of the parallel planes, for instance, is solved by writing down the coordinates of the original plane in a 3-Dimensional coordinate system and by adding a translation vector (a triplet of numbers) to these original coordinates.

that is not algorithmically determinable. In §3.4 we will discuss Gödel's useless metaphysical theorem.

Darwinian math and signals, discovery and invention

This list of questions about differences between 'natural' and 'human' math can be enlarged at will. But even if all the objections can be met, do we indeed know that there is no difference between Darwinian mathematics and human mathematics? Hence, how can we determine what counts as doing math in *the same way*? What *analogy* are we looking for to answer this question positively?

We will come back to this question after we have discussed human mathematics (§3.3). Before doing this, there is one big question about mathematics that might be clearer when asked outside the field of human mathematics: are mathematical results discovered or created? Maybe this question becomes clearer when asked outside of the narrow field of human mathematics.

The first thing to notice about this question is that normally we discover or create *things*, not signs. What does it mean to discover or create a sign? If mathematics is concerned with *signs*, then the question seems to be misguided right from the start. Or *can* we discover signs? A deer 'discovers' that yellow is a sign of a lion, a bird 'discovers' that the polar star is a sign of the north. 'Hence, it seems when we discover a sign, we discover immediately what it is a sign of, what it is about: the thing behind it. When we discover a sign, we discover the thing behind it. At the same time this discovery is a form of creation, because something is *taken* as something.'

No, in this way it seems that signs are representations of objects that create an idea of the object in the receiver. Darwinism denies the existence of ideas; how can a mere idea, a pure mental entity, be of any use to a receiver? The deer is not mentally representing the lion when it sees the latter, it runs away! The bird is not mentally representing the polar star, it corrects its flight direction towards it.

The sign 'yellow' thus *directs* the deer away from the lion, the sign 'polar star' *aligns* the bird to the north. By directing and aligning animals these signs, however, do articulate the environment; they revelate a *difference* in the world between an area with lions and one without them, between north and south.

Now let's look again at the example of the small desert creatures with their 13- and 17 year cycles. They take the sun (or the rain, because it rains only once every year in the deserts in which they live), maybe because of some temperature effect, as a sign of a new year. They are of course not mentally representing the sun, but come out of the sand as it appears again. In this way the (temperature) sign directs these creatures, and at the same time marks the *difference* between sun and not-sun, between one year and another.

But this example was important because these creatures didn't show up every year, but only in a *prime* number of years. Where is this number of years? Is it part of the temperature sign? Or don't we rather need two signs, one sign of the sun ('come out of the desert sand!') and another that keeps track of the number of years that have already past, one qualitative the other a quantitative sign? But how do these signs then cooperate? Do we need a third sign (signal) for that (and a fourth, a fifth etc. to tell this third that it now has to connect these two signs etc.)?

Maybe we can see things clearer if we don't concentrate in this example on the sign as such, but on its manipulation by natural selection. The DNA of those cicadas, programmed to let them arise every 2, 3, 4 or 5 years or one of their multiples (i.e. 6, 8, 9, 10, 12, 14, 15, 16 years), get literally crossed out due to predators whose DNA is programmed to show up every 2, 3 ,4 or 5 years. Those pieces of DNA die out, those signs die out. What remains are the pieces of DNA that program their creatures in such a way that they take a prime number of temperature drops as a signal to get out of their hibernation.

Hence, there seems to be only *one* sign: *one* sign to come out of the sand after a prime number of years. The mathematics comes along with the *sign as sign*. In that sense it is already there, it is 'a priori'.

How can this be? We saw that the sign revelated a difference between one year and another. The sign cuts the time into discrete years. Or rather: the sign itself is discrete (from lat. *dis-cernere*, Greek *krinein*): this year and not-that year, lion and not-lion, north and south. The sign is a cut. The discreteness of the cut brings along arithmetic, since arithmetic deals with the discrete as discrete. But how, exactly? We postpone this question to the section on mathematics and science (§3.42).

Thus the question: is mathematics found or created, are prime numbers discovered or man-made? might be answered: it comes along with the sign as sign. The first option suggested that the prime numbers are already there in *nature*, the second that the prime numbers are an *artificial* structure. Seeing that mathematics deals with *signs*, this question returns as the question whether signs natural or artificial? The signs arise *naturally* in the process of evolution, but at the same time nature seems indifferent to its own mathematization in this *creative* process: the sun doesn't care that its continuous revolution is 'taken' as one, two, three or four etc. revolutions (i.e. *as revolution* in the first place). It seems the sign cuts across the difference of natural and created.

A lot of questions remain unanswered here: what is arithmetic in relation to the rest of mathematics? Just a branch of mathematics amongst others, or central to all math? Does not geometry, definitely a branch of mathematics, deal with the continuous instead of the discrete? Can this be reduced to the discrete?[41] And what would reduction mean in these cases?

A discussion of 'artificial' human math may shed some light on these questions.

§3.3 Human mathematics

It is metaphysical common ground to say that what something *is*, is not determined by the way it came into existence or became known. In physics this motto holds true too, we don't need to know that Copernican was appointed by a sun lover at the Polish court who wanted to reform the calendar, to understand the Copernican system. Yet for Darwinism a big part of the answer of what something *is*, how a complex thing *works*, is how it evolved out of simple elements. If we want to ask about human mathematics in a Darwinian way, we'd better start from the way we and humans before us learned it.

Every child in the western world, whether he likes it or not, is trained from an early age on in mathematics. In the Dutch schools every morning one and a half hours are reserved for *math*, then there is time for a break, after which there was time for *language*: reading, writing etc. Without explicitly defining each discipline, math is contrasted by language. This first math concentrates on arithmetic. You spend the first years learning the tables of multiplication by hearth – which is more of a memorization task than true math; later you learn some basic manipulations using these tables, such as calculations with fractions and long division. In these years at primary school math is equal to *calculating*, formal proofs are not given.

This primary school math is simple and elementary, and occupies itself mainly with numbers. But is it simple, because we are trained in it for such a long time? Is it just a prelude to serious math later on, a prelude to see whether it suits you or not? Or is it still basic for all math that follows later on?

At high school math is not only distinguished from *language*, but from *natural sciences* too. These sciences use math, yet aren't math themselves. Of these sciences a definition is given: physics is about nature insofar as it is non-living and its processes are reversible, chemistry is about nature insofar as it is non-living and its processes are irreversible and biology is about life. As wrong and unsatisfying these definitions are, then what is life, are – due to the second law of

[41] Nowadays, geometry may be reduced to analytic geometry, i.e. arithmetic, but the Greeks saw numbers as lines, so they reduced arithmetic to geometry. Are we better than the Greeks?!

thermodynamics – not all physical processes just as irreversible as chemical ones etc., one would wish to find a definition of math. At the same time the topics of math start to diverge more and more, arithmetic is generalized in algebra, which is distinguished from geometry (stereometry), probability theory, calculus and trigonometry. Not only what math is remains *undefined* here, but also what the principle of division in these branches is and what these branches themselves are. This is the more unsatisfying, because elements of one branch are used in that of another, algebra in trigonometry, trigonometry in geometry, geometry in stereometry, probability in algebra etc.; all branches use arithmetic.

Thus Spengler seems to make a good point, when he writes:

> 'If mathematics were just a science as astronomy or mineralogy, one would be able to define its object. One is, however, unable to do this, not now and not in the past.'[42]

Spengler's words suggest a clear *limit* to our question what is math. Spengler suggests that, although mathematics is a science – traditionally even the most exact of all – its exact object is unclear. That sounds paradoxical: the science that promises the clearest knowledge has an unclear, dark object. And it seems blatantly false too! Numbers are a clear example of mathematical objects and they do not seem to be dark at all: everybody learns from an early age on how to manipulate them. It takes much more time and effort to learn the laws of the movement of the heavens, and even more to learn about the structure of the different atoms and their chemical bonds that constitute minerals.

'Ok, that may be true; the manipulation of numbers is indeed easier than the manipulation of the laws of the movement of the heavens or the chemical formula's, but number itself is much darker than stars or minerals.' This remark seems to be directed at the *origins* and *existence* of number. And indeed, 'one could call the existence [Dasein] of numbers a mystery. (p. 76 idem) [43]" .' But then the existence of natural laws is a mystery too, and we don't buy anything for that.

Thus, if one reads Spengler's words as if he means that the objects of mathematics are unclear, his remark clearly is false. But indeed, he doesn't say this; he writes, we cannot *define* its 'object'. The problem is not the mathematical 'objects' as such, but to define, to delineate what counts as mathematical 'object'. Astronomy deals with stars, mineralogy with minerals, but mathematics... with mathematical 'objects'... But what are they? The Greeks knew geometry and

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[42] Oswald Spengler, *Untergang des Abendlandes, Vom Sinn der Zahlen*, p.82.

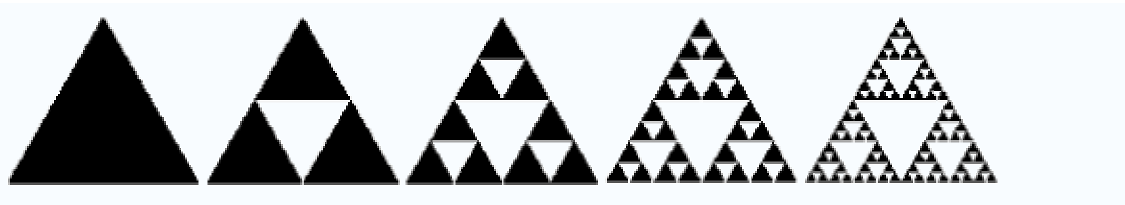
[43] 'That is why mathematics is [considered to be] something holy, that often originates from religious circles – Pythagoras, Descartes, Pascal – and that is why the mysticism of holy numbers, the 3, 7 and 12, form a characteristic feature of all religions.' Spengler, Untergang des Abendlandes, p. 865.

arithmetic as mathematical sciences, and sometimes included music, astronomy, geography and optics. And in our time there are many more branches of math. What do they have in common? As is well known, Descartes had the vision of *universal* mathematics, 'a science [in which is] contained everything on account of which others are called parts of mathematics.'[44] But what is it? And how can it be so universal?

Hence our initial survey of human mathematics, has already brought up a lot of questions, about origins, status, unity of its 'objects', about the relation with different sciences and disciplines. We will first discuss the actual practice of a mathematician (§3.31), then we will look at the history of mathematics (§3.32). That will make it possible to easily dismiss the traditional philosophies of mathematics (§3.33). After that we will discuss the relation of math and science (§3.42) and finally the relation of math and language (§3.5).

§ 3.31 The practice of the Mathematician

Let's look at an example of a mathematical structure and the algorithm to generate it to better understand what mathematicians are actually doing. 1) Imagine an equal sides triangle (a triangle with angles of 60 degrees, first image). 2) Shrink the triangle by 1/2, make three copies, and position the three copies so that each triangle touches the two other triangles at a corner (image 2). 3) Repeat step 2 with each of the smaller triangles (image 3 and so on).



[44] See AT 10, p. 377. Does this universal mathematics only subsume *properly* mathematical fields? I think not, in the text above Descartes, after saying that everybody could easily distinguish mathematics from non-mathematical matter: 'all those matter were only referred to mathematics in which order or measure are investigated, and that it makes no difference whether it be in numbers, figures, *stars*, *sounds* or any other object that the question of measurement arises. I saw consequently that there must be some general science to explain everything which can be asked concerning measure and order not predicated of any special subject matter. This, I perceived, was called 'Universal Mathematics' (ibid., *my emphasis*).' Just as in the Medieval period astronomy and music were 'mathematical sciences' (actually *scientiae mixtae*), so Descartes subsumes under universal mathematics also 'applied' mathematics such as mathematical physics or music theory. It is to be noted that 'physics' (nature) and 'music' changed their meaning due to Descartes': they *are* only insofar they are mathematical (§3.42).

We can proof a lot of things about these figures and the algorithm to generate them.

I) For instance that we can continue with this process indefinitely (for give me a final step, and I'll apply step 2 and 3 again).

II) Or that in each subsequent step the black area becomes less and less (I proof this in the transformation of the first figure into the second figure, and then add that in the next transformation the same thing happens to each of the smaller triangles out of which the next figure is composed).

III) I can even proof that the amount of black can become smaller than any fixed amount. The formula for the black area of each figure is: $(0{,}75)^x$. If I want to show that this can become smaller than any fixed amount, I have to show that – mathematically speaking – $(0{,}75)^x$ goes to 0 if x goes to infinity, a standard result of calculus.

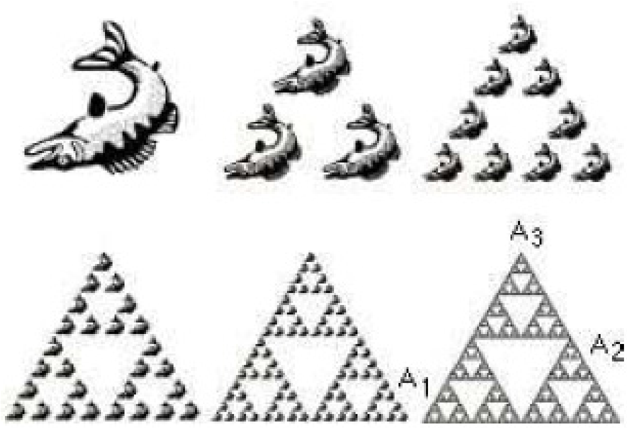
'But I don't understand this; it goes for to quick for me.' Ok, let's try it again: in each step the black area is only 75% of the original figure, hence the second figure has only 75% of the area of the first figure, the third figure has 75% of 75% of the original amount of black, in the fourth 75% of 75% of 75% etc. (some sort of inflation on inflation etc.). If you add enough steps there remains (almost) nothing, just as if you wait long enough to spend your 100 euro – without putting it on the bank – you cannot buy anything with it anymore, due to inflation.

One sees what we are doing: we start with this figure and say more and more things about it that are already *in* it, but which we didn't see as we began. However, once seen and proofed these properties seem to have been there all along. It seems that *as* we constructed this figure, these properties were naturally brought along with it. There is nothing spooky about this. If I) or II) are not spooky, why would the little bit more complicated theorem III) be spooky?

'Ok, that is all very nice; but this is just because you set up the procedure to get these pictures, it is your own construction not *natural* math. It misses generality.'

But look at what Michael Barnsley proofs about this algorithmic process: he shows that this process is independent of the starting shape being a triangle. Barsley even used a fish in the paper in which he proofs this fact, to illustrate this:[45]

[45] See http://wwwmaths.anu.edu.au/~barnsley/pdfs/V-var_super_fractals.pdf (consulted on 27 February 2006). The picture above is taken from the Wikipedia (English version, same date).



Does this general result (the independency of the algorithm of the initial shape) make the figure and the algorithm more *natural*? Does it make it an eternal idea?[46]

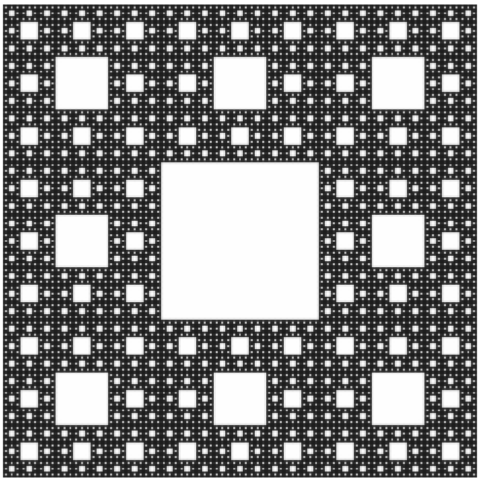
'Ok, so maybe there are some more *general* properties of this object, which I wouldn't have guessed in advance. But still why could we care about this figure? "You proof a lot but you don't proof anything," as I heard a PhD of mathematics once say to his professor, meaning: this is all correct, but what is its use? Only useful math is real, natural math.'

Thus proofs means two things: a proof is correct and a proof is useful, and the first meaning goes back to the first. This explains that, although there are thousands and thousands of mathematicians, there are only a few who have left us 'real' math, 'real' proof, over the centuries. They were the ones that not only found correct proofs, but also *useful* ones.

What does useful mean here? Is it useful for mathematicians or for something else? We saw that the Greeks pursued mathematics for its own sake not for any external use – as is said of modern pure mathematics too – so what could useful mean? It means, useful insofar we learn more and more about the *thing (sign) itself*. 'You mean by generalisation?' No, because by generalisation we don't learn anything more about the *thing (sign) itself*. By generalisation we go further and further away from the original figure and end up with a meaningless abstraction. The proof that the final figure – actually called Sierpinski Triangle – is independent of the initial shape looks like a generalisation (we can use *any* shape to achieve this), but isn't. It teaches us more about what makes the Sierpinksi Triangle *what it is*, not its initial shape but the process of

[46] See Roger Penrose, *The Emperor's New Mind*, Oxford UP, 1999 (1989), p. 124 who (sic!) indeed thinks this is the case 'The Mandelbrot set [the name of a particular fractal] is not an invention of the human mind: it was a discovery. Like the Mount Everest, the Mandelbrot set is just *there*! Likewise, the very system of complex numbers has a profound and timeless reality which goes beyond the particular constructions of any particular mathematician.'

generating it by shrinking it, making three copies and putting them in a special order, makes this figure the Sierpinski Triangle. It learns us furthermore why the natural way to construct it was by choosing the initial figure to be a triangle (instead of a fish), because it corresponds *naturally* to the arrangement of the pieces. Thus by the proof we learn that although the original figure was a triangle, the process of generating it, a process of making a triangle, is what is so special about it. Hence we learn more about this figure and not about some abstract generalisation. At the same time, however, this proof opens up the way to ask about different construction processes. Why not try squares instead of triangles? For instance by copying 9 of them, removing the middle one and repeating the process indefinitely. This gives the Sierpinski *carpet*:[47]



A lot of questions are now natural to ask. Is the outcome of this figure also independent of the original shape of the figure? Is its black area also going to zero? (Yes, but after more iteration.) And can we also do a thing like this with heptagons or hexagons? And how many copies do we need to make in these cases, how to arrange them? Etc.

So a useful proof is not a generalisation, but one that learns more about the thing (sign); due to this proof we may see, however, new ways of exploring that may point in a more general direction.

'But still I don't see the use of these figures.' So did many modern mathematicians, because although they often say they pursue mathematics for itself, they still justify themselves

[47] See http://en.wikipedia.org/wiki/Sierpinski_carpet (consulted 27 February 2006).

by saying that even what we now regard as pure math will one day be useful.[48] And useful means here of course, useful for practical calculations, *useful for science*. The above figures were already described at the beginning of the twentieth century, but were basically forgotten until Mandelbrot rediscovered them in the 1970s, baptising them fractals. This time mathematicians were interested, just as well as scientists, because Mandelbrot added to his rediscovery the claim that fractals might be of enormous importance to describe lots of shapes in nature. He thought of the branches of trees (the branches are little trees, hence 'copies' of the original tree), snowflakes, systems of blood vessels and many others things. See for instance the picture of a piece of broccoli below:[49]

As can be seen from the picture, in the sub structures of the stalk the main structure is repeated again and again.

However, recently the fractal hype declined again. That may seem strange, because they look very useful to describe these natural shapes. Of course in nature, due to the final size of the atoms, a fractal can never be realised into infinite detail, but that does not really limit the application of fractals to describe nature. And the other way round: physics doesn't spoil math. An atom is about 0.2 nanometre in diameter that does not mean we don't understand what 0.002 nanometre would be. Neither does this mean that in some Platonic heaven ideal fractals exits, nor that in nature fractals only exist as approximations, as images of the true platonic fractals (by

[48] See Nikolai Lobatchevsky, one of the founders of Non-Euclidean geometry, in N. Rose *Mathematical Maxims and Minims*, Raleigh NC: Rome Press Inc., 1988'There is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world.' The *tone* of Henry John Stephen Smith's [1826-1883] toast 'Pure mathematics, may it never be of any use to anyone,' only endorses this. In H. Eves *Mathematical Circles Squared*, Boston: Prindle, Weber and Schmidt, 1972.
[49] Taken from the Wikipedia, article 'fractals', 26 February 2006.

some sort of participation). We come back to this problem when we speak of the relation of math and science. There we will also discuss why fractals are no longer hyped.

In sum, we saw that the actual mathematical practice has nothing to do with listing axioms and deriving conclusions from them in a logical way. Mathematical structures are so rich that they can be addressed in many ways. These cannot all be foreseen. The question about the structure has to be answered by what is already present in the figure, which is often very difficult. But once found, the perhaps unexpected answer is clear, although of course mistakes can be made. This answer is communicated by a formal proof, the axioms of which were often not there before the proof is given: the way of addressing a problem in the first place determines which axioms are useful. Not everybody has to understand this proof. There is quite some truth in the proverbial wisdom 'in mathematics you just see it or you don't', although training can certainly help a lot. To this person who does not understand, we can repeat the proof again, perhaps in a simpler, a more detailed or an altogether different form (see the way we talked about the proof that the area of the Sierpinski Triangle is zero).

Because of the many ways of addressing a structure lots of different proofs can be given, but many of them are uninteresting. Only interesting, useful proofs are the ones that remain known. Useful is proof that shows what a structure itself *is*. These proofs are not necessarily generalisations, more often a closer look than an abstraction[50], although this closer look may still, as a side effect, give rise to generalisations. In our times, however, usefulness seems to have narrowed down to usefulness for the sciences. It is even the case that lots of math was developed because there were certain problems in physics, for instance infinitesimal calculus. We take a closer look at the usefulness of math in the section on math and the sciences (§3.4).

§ 3.32 The history of mathematics

Spengler suggests that there is not one form of mathematics in history, but many that differ in many regards from another:[51]

[50] See also David Hilbert 'In dealing with mathematical, specialization plays, as I still believe a more important part than generalization Bulletin of the American Mathematical Society, vol., 8 (1902), p. 445).'
[51] See also p. 84 'The mathematic, then, is an art. As such it has its styles and style periods. It is not, as the layman and the philosopher (who is in this matter a layman too) imagine, substantially unalterable, but subject like every art to unnoticed changes form epoch to epoch.'
This is strong contrast with David Hilbert 'Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country,' in H. Eves *Mathematical Circles Squared*, Boston: Prindle, Weber and Schmidt, 1972. In our time, at the end of history, due to a narrowing of what math is, Hilbert may be closer to the truth than Spengler. See the end of the paragraph on math and science.

‘Even though we, West Europeans, may impose our own scientific concept of number violently on the things mathematicians in Athens and Baghdad dealt with, this at least is certain, that the themes, intentions and methods of this eponymous science were completely different over there (p.82).’

And indeed if we compare just the *tone* of what Greek and modern mathematicians say about their discipline and its use, this becomes extremely apparent. Just a few examples: Aristotle says that math was invented by the Egyptian priests at the moment all practical necessities were taken care of and there was time for disinterested thought.[52] A student asks Euclid at the end of his first lesson in geometry what advantage he will gain by learning such things, whereupon Euclid summons a slave and exclaims: ‘Give him three obols since he must need gains out of what he learns.’[53] Archimedes – the greatest mathematician of Antiquity – died by the hand of a soldier as he ‘dreamt of his circles’.[54]

Compare this with the greatest mathematician of the modern times, Gauss, who seems to have more in common with the soldier who killed Archimedes than with Archimedes himself:

‘It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment. When I have clarified and exhausted a subject, then I turn away from it, in order to go into darkness again; the never-satisfied man is so strange if he has completed a structure, then it is not in order to dwell in it peacefully, but in order to begin another. I imagine the world conqueror must feel thus, who, after one kingdom is scarcely conquered, stretches out his arms for others.’[55]

Or take the lecture that Hilbert, one of the greatest ‘conquerors in mathematics’ of the twentieth century, held before the international congress of mathematics in Paris in 1900, at which he formulated his famous 23 problems that 20th century mathematics would have to solve (and of which 20 are now solved), a lecture that can best be read as a *plan de campagne*.[56]

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[52] Aristotle, *Metaphysics* 981b1 ‘Now that practical skills have developed enough to provide adequately for material needs, one of these sciences which are not devoted to utilitarian ends [mathematics] has been able to arise in Egypt, the priestly caste there having the leisure necessary for disinterested research.’
[53] A story reported by Stobaeus in his *Eclogues* II.31. See *Classics of Mathematics*, Ronald Calingher (ed.), Prentice Hall London 1995, p. 109.
[54] Idem, p. 131 ‘The roman soldiers feared Archimedes for the devices he invented to defend the city on land and from attacks from the Roman navy. These devices probably included battering rams, catapults, cranes, a compound pulley to move ships onshore easily, and perhaps burning (paraboiloidal) mirrors. According to Livy, Plutarch, and Valerius Maximus, he was working on a geometrical problem when a Roman soldier surprised him. He either asked the soldier for more time to complete his proof or ordered him not to disturb his drawing in the sand board. (...) Angered, the soldier killed him.’
[55] Gauss, letter to Bolyai, 1808.
[56] See Hilbert, *Bulletin of the American Mathematical Society*, vol., 8 (1902), p. 440 ‘As long as a branch of science offers an abundance of problems, so long is it alive; a lack of problems foreshadows extinction or the cessation of independent development. Just as every human undertaking pursues certain objects, so also

But not only the tone is different, also the problems and the methods. Greek mathematics had three classical problems: the quadrature of the circle, the trisection of an angle and the doubling of the volume of a given cube. They solved these problems numerous times, but not in the way they wanted it: with a straightedge and a compass. We would say: you already found a way to solve these problems, why the restriction that you have to solve these problems with only a straightedge and a compass? On the other hand, the Greeks wouldn't understand the fact that we try to *deduce* very evident propositions in elementary calculus, using set theory (remember Russell finally solving 1 + 1 = 2 at the *end* of the first volume of his *Principia Mathematica*!)[57]

Finally, also the 'concepts' and 'objects', the *signs* that are used, differ a lot during history. Look for instance at the sign 'zero'. There is not one 'zero' about which we learn more and more.[58] The Babylonians knew 'zero' as an empty place marker. In their early 60 number system, '60' and '3600' looked the same, they just wrote the symbol for 60 in both instances, leaving some space blank in the second case (i.e. '60' versus '60 '). Later they added a dot to mark the blank space (i.e. '60' versus '60·'=3600). They didn't use this dot separately to write down a single zero, the first to do this were the Indians. They could write as the solution of the sum '8 - 5- 3' the numeral 'zero'. Later they could write decimal approximations of fractions (i.e. $^1/_5$ as 0.2). The classical Greeks didn't know the zero at all. Only in the later Hellenistic age, in their exchange with Indian astronomers, the Greeks used – reluctantly – these Indian numbers to ease calculations, only to convert them back into the ratios of their own number system after the calculations were done. The Arabs were the first to introduce our modern notation for the 'zero'. Both the Arabs and the Indians didn't know negative numbers. In Medieval Europe the zero became the neutral (neither even nor uneven) element between positive and negative numbers on the number line. In Medieval arithmetic books the rule 'number times 0 = 0' became part of calculus. In the Renaissance the zero point was discovered in the new perspective form of painting. The zero point is that point at the horizon at which all (infinitely many) lines come together. This special relation of the 'zero' with the 'infinite' became later more explicit in the prohibition to divide a number by zero (i.e. '1/0 = ∞'). With the development of infinitesimal calculus, it became clear that this prohibition doesn't hold for the number zero itself, zero can in

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mathematical research requires its problems. It is by the solution of problems that the investigator tests the temper of his steel; he finds new methods and new outlooks, and gains a wider and freer horizon.'

[57] See Spengler, *Untergang des Abendlandes*, p. 114 'Especially the most 'evident' propositions of elementary calculus – that for instance 2 times 2 is 4 – become [in our times] (...) problems whose solutions can only be achieved by deductions out of set theory, which in all its details hasn't been succeeded yet – something Plato and his contemporaries would absolutely consider madness and a clear sign of a total lack of mathematical talent.'

[58] See, *Zero – the history of a number*, Penguin 2001.

certain special cases be divided by itself (i.e. ‘0/0’) and the result can be 0, ∞ or any other number. The latest set theoretic and algebraic theories see zero just as the neutral element of *any* group under the operation of addition.

Is there one ‘zero’ about which we have learnt more and more? Is their one ideal ‘zero’ behind this whole history? Of course not! Indeed, we can ‘impose our own scientific concept of number violently on the things mathematicians in Athens and Baghdad dealt with’, as Spengler said, but have we indeed learnt more about the same Greek and Arabic numbers? And do we understand why the Greeks were so reluctant to accept the number zero? I don’t think so.

How to understand the difference between for instance Greek mathematics and our own mathematics? Spengler gives a hint, when he says that for the Greek math deals with *corporeal bodies*[59], whereas ours deals with *dynamical functions*. That is why the Greeks are so reluctant to accept the number ‘0’, because it is not a body. That is why the Greeks were shocked to learn about irrational numbers, because ‘the irrational, in our way of expression the use of infinite decimal fractions, meant a destruction of the (...) corporeal order.’[60] That is why the Greeks speak of *triangular* and *square* numbers.[61] That is why the Greeks don’t know any higher powers than the third power. That’s why the Greek geometry knows only 3 dimensions, because there is no 4-Dimensional *body*.[62] That’s why, contrary to popular belief, Greek geometry doesn’t deal with (astronomical) space, but with ‘small manageable, handy figures and bodies.’[63] That is why the Greeks demanded straightedge and ruler proofs, because in these cases the figures and could be constructed out of real corporeal elements. And that is finally why the Greeks, the Pythagoreans above all, saw a clear relation between number and our own bodies, because they already understood numbers as ‘σώματα’.[64]

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[59] See Spengler, *Untergang des Abendlandes*, p. 87 ‘The mature classical mind could only understand mathematics – in agreement with its total feeling of the world – as the doctrine of the relations of sizes, measures and shapes of *corporeal* bodies.’

[60] Idem, p. 112.

[61] 3, 6, 10 and 15 are for instance triangular numbers, 4, 9, 16 and 25 square numbers. Triangular numbers can be arranged as a triangle, square numbers as a square (see below), although maybe the word arrange is the wrong word to describe this. The numbers themselves are squares or triangles, not because we subjects arrange them that way.

3	x	4	xx	6	x	9	xxx
	xx		xx		xx		xxx
					xxx		xxx

[62] Spengler, *Untergang des Abendlandes*, p. 90 ‘Plato and Archytas speak about area and body numbers when they mean our second and third powers, and it is clear that the concept of higher whole-numbered powers don’t exist for them.’

[63] Spengler, *Untergang des Abendlandes*, p. 113. See also footnote 29.

[64] Spengler, *Untergang des Abendlandes*, p. 112 ‘The Greek mathematicians even use the word σωμα for their bodies (...). That’s why the classical, whole, corporeal number instinctively seeks association with the

We on the other hand implicitly understand numbers and mathematics in general as dealing with functions. This becomes most explicit in set theory in which numbers are reduced to their function within a group of operations. The 'zero' is that element which projects a number unto itself under the operation of addition ($x + 0 = 0$), and unto zero under the operation of multiplication (x times $0 = 0$). The 'one' is that element which projects a number unto the next one under the operation of addition, and unto itself under the operation of multiplication. Because of this functional understanding of mathematics, we don't have problems with the zero, the irrational or even the imaginary numbers (see footnote 28). That's why we know geometries of more than three dimensions, because in our analytic geometry spatial points, area's and bodies are just sets of three independent numbers, that – once their 'optical character of a sectional plane of coordinates in a intuitive imaginable system' – gradually faded, could easily be enlarged to more dimensions (i.e. sets of more than three independent numbers). That's why we don't have problems with the 'non extensional entities', the infinitesimals that are fundamental to the calculation of integrals and differentials etc.[65]

In the end all these differences Spengler points out go – according to him – back to a fundamental difference between the *Ursymbols* of the Greek and the modern culture, the Ursymbol of the *Apollonian shining corporality* in the here and now versus the Ursymbol of the *Faustian will to power* that strives towards ever further and further times and spaces. Darwinists can of course not accept these symbols found by the 'morphological genius' of Spengler. It would mean reducing the forms of mathematics to a deeper *metaphysical form*.

Darwinistically speaking, we can accept an evolution of number systems: the ones that aren't used anymore simply died out. The main reason for dying out is probably the disfunctionality: Greco-Roman numerals are just a nuisance to do anything serious with. The

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generation of corporal humans, of σώμα. The number 1 is hardly experienced as a real number. It is the αρχη, the prime matter of the number line, the origin of all real numbers and thus of all quantities, all measures, all things as such. Its numeral symbol was in the circle of the Pythagoreans (...) at the same time the symbol of the womb, the origin of all life. The 2, the first *real* number, which doubles the one, became consequently a relation with the male principle, and its symbol was an imitation of the phallus. Finally, the *holy 3* of the Pythagoreans, denotes the act of the joining of man and wife, the procreation and its symbol was the joining of the first two.'

[65] Spengler would even argue that the initial problem with infinitesimals was that they were understood as Greek numbers, as extensional bodies. Once they were understood as functions, the problem was gone. See page 117 'Until the eighteenth century did Euclidean-popular prejudices obscure the meaning of the principle of differentiation. Even if the initially natural concept of the infinitely small is used as carefully as possible, a little moment of classical stability will stick to it, the appearance of a quantity, even though Euclid wouldn't have understood, wouldn't have acknowledged it. The zero is a constant, a whole number in the linear continuum between -1 and +1; it damaged Euler's analytical investigations that he – and many after him – held the differentials to be zeros. Only the concept of the limiting value that was finally elucidated by Cauchy destroyed this relic of the classical number sense and made the infinitesimal calculus a system that is free from contradictions.'

Hindu-Arabic numerals are so successful that they have become the main number system world wide.[66] Yet functionality alone seems not enough to describe the evolution of mathematics. Spengler's description of Greek mathematics points to something non-immediately evolutionary. When contrasted with our modern math, it seems that the character and problems of Greek mathematics flow from an *analogous* source: the implicit understanding of mathematical signs as corporeal bodies. How can this analogy arise? Through the *word* 'soma'? Is mathematics then bound by language? We will come back to these questions when we talk about math and language (§3.5).

Having now some acquaintance with the practice of the mathematician, who doesn't deal with abstractions nor with meaningless identities, and with some knowledge the history of mathematics, we can turn very briefly to the different philosophies of mathematics and their many problems.

§3.3 The Philosophies of Mathematics

There are four main schools in the so-called philosophy of mathematics: Platonism, logicism, intuitionism and formalism. They either try to account for the certainty of our mathematical knowledge as the first and the fourth, or try to make this knowledge more secure as the second and the third. All but Platonism arose from the foundational crisis in mathematics at the beginning of the twentieth century, and all but Platonism suffered major drawbacks as Kurt Gödel, a self confessed Platonist, proofed his famous theorem in 1931. Today many mathematicians will (still) admit, if forced to answer, that they are Platonists too. Yet there are many problems with Platonism. We will speak briefly of Platonism first, then of the other schools and finally of Gödel's proof.

Platonism

Platonism is the theory that mathematical objects have an independent existence. Independent of what? Independent of nature and (possibly) independent of the mind. To see these objects man

[66] See H.W. Eves *Mathematical Circles Squared*, Boston: Prindle, Weber and Schmidt, 1972, p. xxx 'One is hard pressed to think of universal customs that man has successfully established on earth. There is one, however, of which he can boast the universal adoption of the Hindu-Arabic numerals to record numbers. In this we perhaps have man's unique worldwide victory of an idea.'

needs a special intuitive organ.[67] Whether Plato was himself a Platonist in this sense is doubtful; we will, however, not pursue this question here. Platonism originates from Aristotle's understanding of the Plato's ideas as universal independent. The classical locus where Aristotle. It is worth quoting this passage *in extenso*, because it sheds light on the relation of science and mathematics that we want to dwell upon later in this chapter:

'The next point to consider is how the mathematician differs from the physicist. Obviously physical bodies contain surfaces, volumes, lines and points, and these are the theme of mathematics (...). Now the mathematician, though he too treats of these things (viz., surfaces, volumes, lengths and points), does not treat them as the limits of a physical body; nor does he consider the attributes indicated as the attributes of such bodies. That is why he separates them, for in thought they are separable from motion, and it makes no difference nor does any falsity result if they are separated. Those who believe the theory of the forms, though they are not aware of it; for they separate the objects of physics, which are less separable than those of mathematics. This is evident if one tries to state in each of the two cases the definitions of the things and of their attributes. Odd, even, straight, curved, and likewise number, line and figure do not involve change; not so flesh and bone and man – these are defined like snub nose, not like curved. Similar evidence is supplied by the more physical branches of mathematics, such as optics, harmonics, and astronomy. These are in a way the converse of geometry. While geometry investigates physical lengths, but not as physical, optics investigates mathematical lengths, but as physical, not as mathematical.'[68]

Hence, Aristotle holds against the Platonists that they do not realise that basically what they are doing is no more than engaging in a process of separation in thought, and furthermore that they choose the wrong things to separate. The mathematician can only study surfaces, volumes, lengths and points of *physical* bodies, but not *as* physical bodies. The physicist can also study surfaces, volumes and lengths but not as mathematical, he studies them as physical (that is, subject to change). There doesn't lay an idea of these physical bodies behind them.

Aristotle does seem to thinks that mathematicians deal with objects that are the result of an abstraction in thought.[69] These abstractions are, however, bound by *language*. We can *speak* of a man *as* undividable, then we are in the realm of arithmetic, and we can also *speak* of a man *as* dividable, then we are in the realm of geometry. This becomes clear when one reads Aristotle

[67] See for instance Charles Hermite (1822 – 1901), a French mathematician after whom the Hermite polynomials, Hermite normal form, Hermite operators and cubic Hermite splines are named. 'There exists, if I am not mistaken, an entire world which is the totality of mathematical truths, to which we have access only with our mind, just as a world of physical reality exists, the one like the other independent of ourselves, both of divine creation.' In *The Mathematical Intelligencer*, v. 5, no. 4.
[68] Aristotle, *Physics B2*, 193b23-194a12.
[69] See for instance Jonathan Lear, *Aristotle's Philosophy of Mathematics*, The Philosophical Review, XCI, No. 2 (April 1982).

himself too.[70] Nevertheless after Aristotle it became a commonplace to think that mathematical structures are the result of abstractions in thought, without taking notice of the abstraction being bound by language.

Locke, and Berkeley in his foot steps, ridiculed this idea of *abstraction* in math. What triangle is the ideal triangle if we have to abstract from any *specific* triangle, if it is not actute-angled, not obtuse-angled, not isosceles, not equiangular, not big not small, what is it? What is this entity without properties? What is this idea???

Or, looking at the history of mathematics, we might ask: what historical triangle is the ideal triangle? The one of Egyptian land measurement, the Greek Pythagorean triangles of the triangular numbers, the Greek triangle of Euclidian geometry, the triangle of Renaissance perspective drawing, the triangle of analytic geometry that is defined by the triangle inequality, the generalised triangle of modern Non-Euclidean geometry???

Despite all these unsurpassable problems, Husserl later took up the challenge to understand what the ideal triangle is, yet admitting that the concrete triangle we imagine is always a big/small, actute-angled/obtuse-angled one. He tried to understand the different properties of the *specific* triangles as the different acts of a subject constructing this triangle in his intuition, an act that determines the size of the triangle, another that determined the colour, another the type of angles etc. The triangle that is intended 'in'/'by' all these subjective acts is, however, itself an eternal *eidetic object*. But despite Husserl's many distinctions, the relation between the different acts and the access to this eidetic object remain unclear.

At the end of his life Husserl wrote *The Crisis of the European Sciences*. A famous part of this work is the chapter entitled *The origins of the Geometry* in which he tried to understand how there was a time before people knew triangles, how the timeless ideal triangle came to be and what it is supposed to be. However, he didn't take into account that there hasn't been one

[70] See Aristotle, *Metaphysics*, 1077b14-1078a28 'For just as there are many *statements* about things merely *as* moving apart from the nature of each such thing and its incidental properties (and this does not mean that there has to be either some moving object separate from the perceptible objects, or some such entity marked off in them), so in the case of moving things there will be statements and branches of knowledge about them, not *as* moving but merely *as* bodies, and again merely *as* lengths, *as* divisible and *as* indivisible but with position and merely *as* indivisible. So since it is true to *say* without qualification not only that separable things exist (e.g., that moving things exist), it is also true to *say* without qualification that mathematical objects exist and are as they are *said* to be (...) So if one posits objects separated from what is incidental to them and studies them as such, one will not because of this *speak* falsely any more than if one draws a foot on the ground and calls it a foot long; the falsehood is not part of the premises. (...) A man is one and indivisible *as* a man, and the arithmetician posits him *as* one indivisible; the geometer, on the other hand, studies him neither *as* a man nor *as* indivisible, but *as* a solid object [*my emphasis*]'. See also footnote 91.

unique triangle in history as we just described. He can be called a solipsistic Platonist, just as the intuitionists.

Intuitionism:

Intuitionism is almost synonymous with the work of the Dutch mathematician Brouwer and his rejection of the principle of the excluded third. But intuitionism was more than that: it was the desire to reformulate mathematics in such a way that the only allowed mathematical ‘objects’ were the ones that could be constructed by a subject in his (private) *intuition*. In this sense one can call Kant, the Neokantians and even the founders of modern number theory, Dedekind, Cantor amongst others, intuitionists[71] because they thought that man constructs the mathematical concepts. The fact that for Brouwer the intuition in which mathematical structures are constructed is *temporal*, makes the bond with (Neo)Kantianism even stronger

But we don’t need in the sense of a special intuition, spatial or temporal, to grasp mathematical structures, if only is clear how you can manipulate them. For instance, you don’t need an intuition for the imaginary number unit i, defined as “i times i = -1”. Although sometimes a figure, like the one in footnote 28, can help to better grasp the way to manipulate these units. A multiplication with i means a rotation over 90 degrees. These figures are spatial, but also temporal because a multiplication by i is a counter clockwise rotation, a multiplication by –i a clockwise rotation, hence the + or – sign determines the *direction* of the rotation which is not in the spatial figure as such). But the spatial and temporal character is already there in the imaginary numbers themselves too. Take for instance the sign 12i. As said earlier, the first number 1 is a 10, whereas the second is a 2 – the *place* value system. Our *direction* of reading this symbol is from left to right: we read 12i and not 21i – the *temporal* direction of the system. Both the sign and the figure (which is in the end a sign too) hence have a spatial-temporal side, but this has nothing to do with a spatial or temporal *intuition*, this is the spatial-temporal side of the *sign itself*.

One could call intuitionism subjective Platonism. Just as Platonism scorned the use of natural language, Brouwer disliked natural language.[72] The same arguments against Platonism apply to intuitionism. Moreover, we saw before as we spoke of proofs that no personal intuition can warrant that your results are all right. You can have the intuition that 2 + 2 is 5, but the

[71] See for instance Dedekind’s ‘What are and what should we care about numbers? (‘Was sind und was sollen uns die Zahlen’) in which he tries to lay the foundations of number theory. In this little work he constantly speaks about numbers as constructions in our mind.

[72] See Van Dalen 1977. Math itself is for Brouwer a ‘language free activity (p. 21)’; ‘[language itself is but] an imperfect tool for man to share mathematics with each other and to support the man’s mathematical memory [Brouwer 1907].’

teacher will nevertheless correct you time and time again you write 5 down as the solution to this equation. Just as there is no private language, there is no private math.[73]

But the biggest problem with intuitionism is that it tries to reformulate concrete mathematical results. The rule of double negation – used in proofs *ex falso* – was forbidden, because this rule presupposes that you can overlook all possible cases, and these are never given in the *private intuition*. For the same reason the infinite was forbidden in intuitionism. For instance, you cannot proof *ex falso* that there is no highest prime number, by assuming that there is a highest prime number and then showing that this lead to a falsehood, because you cannot check – in your intuition – whether this proof indeed holds for *all* numbers. You simply cannot overlook all possible prime numbers and the number row.

But many sound mathematical results, that are used extensively and successively in the natural sciences, use the infinite and the *ex falso* rule. We even saw that human math differs from 'natural' math in being able to formulate explicit impossibility proofs that always use the *ex falso* rule. Hence 'by depriving the mathematician of the principle of excluded third, is comparable to depriving the astronomer of the use of his telescope or the boxer the use of his fists.'[74]

A philosopher should never try to revise mathematics or a science.

Logicism

Logicism is the idea that all Mathematics is symbolic logic.[75] Its founder is Frege, who set his theory forth in his *Principles of Arithmetic*. In this work he tried to show how arithmetic can be *reduced* to the logical notion of set. That's why logicism is often called set theory. We will give an example of this (flawed) reduction in the section 'on mathematics and number'. Frege thought that these sets and other mathematical concepts existed in a so-called *third* realm apart from the physical and mental realm.[76]

Frege became famous because his work contains the first modern *formal* logic, the first logic that – in strong contrast to the classical syllogistic logic – claimed that its rules were independent of the meaning of its symbols. However, just as the second part of the *Principles of Mathematics* was about to appear Russell, in a letter to Frege, pointed to a fatal flaw in his symbolic language, now known as Russell's paradox. Russell tried to do a better job with his own

[73] See Van Dalen, p. 26 '[T]he writings of Brouwer, especially after 1948, have strong solipsistic characteristics.'
[74] Hilbert, quoted by van Dalen (1977), p. 29.
[75] See Bertrand Russell, Preface of *Principles of Mathematics*, 1903 'The fact that all Mathematics is Symbolic Logic is one of the greatest discoveries of our age; and when this fact has been established, the remainder of the principles of mathematics consists in the analysis of Symbolic Logic itself.'
[76] He thus follows the tripartition of philosophy of German Idealism in Nature, Mind and Logic.

type theory in the *Principia Mathematica* that he wrote together with Whitehead. There was a contradiction in this work too, that forced Russell and Whitehead to formulate their so-called 'ramified theory of types'. In this new theory they had to formulate the axiom of non-contradiction for every new level of the hierarchy of types[77], which meant that mathematics would have an infinite number of axioms, a thought not very welcome to a reductionistic foundationalist program.

The details of this theory need not interest us. Apart from the unnaturalness of having an infinite number of axioms and apart from the unnatural and long proofs of trivial math as 1 + 1 = 2, we already saw that Russell himself gave up his program.

Formalism

Formalism is the brainchild of David Hilbert. It can be summarized in his own infamous words: 'Mathematics is a game played according to certain simple rules with meaningless marks on paper.' These rules are the rules of logic. Hence one could say that formalism is also a form of logicism, albeit without the third realm and sets. Formalism thought math was about deriving conclusions from true axioms. Once the axioms were coded into rules and symbols, one need not care about their interpretation but just apply these logical rules to the initial symbols. After the application of these rules one could translate, interpret the symbols of the result to get something meaningful.

However, in our discussion of the practice of a mathematician we saw the mathematician doesn't work with a fixed set of axioms, and neither does he just deduce theorems from these. New math means new axioms. Furthermore, mathematical signs are not meaningless marks but triangles, fractals, probabilities etc.[78]

[77] Set theory took as fundamental concept mathematics sets, not elements. Elements of sets are always sets too. The origin of both Frege and Russell's problems can be traced to this understanding of sets and elements on the one hand and the possibility of recursive sets on the other. In Russell's paradox (against Frege) this becomes extremely apparent: the set of all sets that doesn't contain itself as an element is indeterminable, just as the hair cutter that cuts every haircutter that doesn't cut his own hair. Russell's own problems with his theory that necessitated him to formulate his ramified theory of types are also the result of the possibility of sets containing itself as element. Modern Zermelo-Fraenckel set theory – never used by mathematicians – just excludes in its definition of a set the possibility of containing itself as a set.

[78] Ironically, Hilbert himself acknowledges this elsewhere too: 'To new concepts correspond, necessarily, new signs.' These signs are furthermore not meaningless marks 'Who does not make use of drawings of segments and rectangles enclosed in one another, when it is required to prove with perfect rigor a difficult theorem on the continuity of functions or the existence of points of condensation? Who could dispense with the figure of the triangle, the circle with its center, or with the cross of three perpendicular axes? Or who would give up the representation of the vector field (...) [etc.]?' Hilbert, Bulletin of the American Mathematical Society, vol. 8 (1902), p. 440.

Formalism was more successful than real logicism, because it is useful to understand the algorithmic working of computers. It is, however, generally believed that Gödel's proof marked the end of Hilbert's formalistic program. We consider this proof next.

Gödel's proof

Gödel's incompleteness proof is a proof that shook the mathematical community. It almost ended single-handedly the search for foundations in mathematics, because it showed that no axiomatic system could proof all true mathematical theorems, hence the search for such an axiomatic system would be futile. This proof is used in all kinds of philosophical speculations about the limits of our knowledge, about the impossibility of a complete science and – most often – about the difference between human minds and computers.[79] Apart from these speculations, Gödel's proof didn't have much consequences; it is an isolated result, which is *never* used in normal math, let alone in the sciences. A mathematician who would say: 'I cannot proof this theorem, because Gödel says that not all math is provable,' is at best being laughed at. Although the wild speculations might suggest otherwise, Gödel's proof simply didn't have any consequences for the evolution of the sciences and their further integration, neither for physics nor for math, neither for computer science nor for neural science. That should at least make as sceptical, whether Gödel's proof is not just a superfluous ornament to the machinery of mathematics.

Gödel himself calls his proof a meta-mathematical one, that is a mathematical proof about possible mathematical theories. This meta-mathematical approach looked like a new approach in his days, but in my opinion lots of math before Gödel were already meta-mathematical, if one wants to use this word at all. For instance, one can calculate many real divisions: 19/2, 19/3, 19/5 … 20/2, 20/3, 20/5… 21/2, 21/3, 21/5… 22/2, 22/3, 22/5… 23/32, 22/3, 22/5 etc, but one can also proof that there is no *prime number* between 19 and 23. The concept 'prime number' is then already 'meta' with respect to all the individual calculations. But normally we don't say that number is a meta-mathematical concept. The same situation occurs in language: part of language are the words 'word' and 'language' themselves, but we don't call these words meta-words. Now if we formulate a theory of language (in language) we could call this meta-language, but even people who study language as such call there discipline linguistics and not meta-linguistics, so we'd better not call Gödel's proof meta-mathematical because it is math about math. Even more so, because I think the proof ultimately depends on the stratification

[79] The first and still most famous paper about this is J.R. Lucas 'Minds, Machines and Gödel' in Philosophy, XXXVI, 1961, p. 112-127. Roger Penrose two best sellers 'The emperor's new mind' and its follow-up 'Shadows of the mind' are for the most part devoted to this problem.

of language, the possibility of *selfreferentiality in language* (hence the 'meta' of the proof is not the 'meta' of math but the 'meta' of language if we want to use the word 'meta' at all).[80]

'Ok, but still, Gödel's proof is not about *some* mathematical theories, it is about *all* possible mathematical theories, that's why the word 'meta' is indeed justified, just as when we don't do linguistics as such, but discuss what is true about any form of linguistics whatsoever.' But the question is whether all math consists out of proofs based on a number of axioms that is fixed in *advance*, as Gödel's proof presupposes. Our discussion of the praxis of the mathematician showed that that is not the case. Hence Gödel's proof is not about all possible math and the word 'meta' is not justified if 'meta' is here understood as saying something about *all* math (if it indeed were then – admitting that it itself is a mathematical proof too – it had to say something about itself too, but despite the proof's fond use of selfreferentiality, it doesn't...). But let's look closer at the proof to better understand what Gödel means by incomplete, improvable and true.

The core of Gödel's proof

Gödel's proof rests on two 'strategies'. The first is a philosophical argumentation, the second a mathematical trick. The philosophical argument is partly hidden, but is essential to the proof. It consists in a distinction between proving something and something being true. Normally in mathematics we identify true with proofed, or on a little reflection, admitting that future mathematicians might proof new theorems, we identify true with what *can* be proofed. Not so in Gödel, Gödel tries to show that there always exists at least one theorem which is true but cannot be proofed. All depends here on what Gödel understands as what is truth and what constitutes a proof. Truth is for him in the end a mental intuition (hence all the arguments that Gödel 'showed' that man and computers are different)[81], and proof is derivability in a system of axioms that are

[80] Gödel seems to admit this as he points out there is a big similarity between his argumentation and the (linguistic!) paradox of the Cretian Liar, already known to the Greeks, who says that 'all Cretians are liars.' This *self referring* statement is true when it is not true, and not true when it is true. (This sentence is by the way only a problem when you believe that every sentence is either true or false. In practice we take a sentence to be true, only later to find out that it is not-true (or the other way round). The funny thing about this sentence is that it doesn't lead to a stable interpretation, just as when you try to catch a mouse, it always runs to the opposite corner.)

[81] In his proof Gödel constructs a sentence that is true, but no computer can derive its truth, because a computer can only make certain mechanical steps out of certain initial states (the axioms) and Gödel can show that the machine cannot reach this sentence of his, hence the computer cannot 'derive' this true sentence out of its true axioms. There is however a *procedure* to construct Gödel's sentence (i.e. the procedure he described in his article). You can of course add this procedure to a computer program. But then Gödel answers that he can construct a new Gödel sentence of this updated computer system, which the new computer system cannot proof. Every program we make suffers from this shortcoming. Gödel can see this in advance, 'we have an *intuition* of that fact', hence we can do something a computer cannot.

fixed in advance. We saw, however, that truth can never be just some intuition, nor is mathematics proofing things from a fixed number of axioms. This notion of proof really has to be understood out of its historical situation: it is Hilbert's notion, we just discussed above under 'formalism'. To Hilbert we already responded that our discussion of the praxis of the mathematician showed that mathematicians don't always use axioms that are fixed in advance, they see something they can proof and afterwards look for the right axioms (possibly new) they can use.

But even granting Gödel's wrong understanding of both truth and proof, his proof is a still a very meagre result. Now the mathematical trick comes in. Gödel lists all the possible combinations of (uninterpreted) symbols that are possible given the (uninterpreted) axioms of symbol manipulation, and gives each of these combinations their own number. The so-called Gödel number. Since these Gödel numbers are also just numbers, they can be part of a string of uninterpreted symbols, i.e. there can be strings 'about' other strings. Some long strings can even contain the steps of a proof of the deduction of a shorter string. Then stepping outside the framework of strings that can be created by applying the deduction rules of symbols, Gödel writes down a string of symbols that cannot be created using these deduction rules. Then he brings the interpretations back in. His string, the so-called Gödel theorem, means: 'there doesn't exist a string that proofs ('contains the deduction') that the Gödel number of a string is (the value) of this string'. This string is true, but cannot be proofed (i.e. deduced using the rules of the system). Hence, in any axiomatic system strong enough to codify arithmetic, there exist true sentences that cannot be proofed. Those systems can thus be consistent ('without contradictions'), but not complete ('all true sentences are provable').

My suggestion is that this is just a linguistic trick, Gödel thinks that he proofs that there is *at least one* thing unprovable, and I think we'd better read: there is *just one* thing[82] unprovable, namely his reflexive sentence. The problem occurs only after the (linguistic) interpretation of the symbols, and *depends critically on the meaning of 'true' and 'proofed'*. The only true and unprovable sentence is one that says nothing about real math, but only about proofs (understood in Gödel's way) themselves. But if one doesn't believe in Hilbert's formalism, Gödel's use of these words doesn't make sense. Our critique on the assumptions of formalism, hence also holds against Gödel: mathematics is not completely axiomatic deductive, there are no fixed axioms, mathematicians don't deal with meaningless marks but with triangles, probabilities etc., true in mathematics means that which *can* be proofed, not intuitive correctness.

The problem is thus that the computer always has a fixed number of axioms (procedures).

[82] And some 'small variations' on it.

§3.41 On mathematics and number

Our pragmatic analysis of human math seems to force us to the conclusion that all mathematical structures, all symbols are on equal par. On the one occasion numbers are used, on another fractals, then again triangles or probabilities etc. Yet mathematicians often express that the numbers are fundamental. Maybe Kronecker's famous words express this common feeling most clearly: 'God created the integers, all else is the work of man.' And many philosophers feel the same. Spengler says it very clearly: 'number is fundamental to all forms of mathematics as *the* given element.'[83]

Are these mathematicians and philosophers just wrong? It seems like that at first, because modern mathematicians have tried to get rid of number altogether.

The first, naïve, attempt at this starts arguing from the fact that there are different number systems, for instance the Roman and the Arabian-Hindu. Because there are different number systems, number cannot be fundamental. 'But that is of course plainly wrong; you should look at the numbers behind the numerals. Those are still fundamental.' This argument looks similar to saying that behind the English word 'bread' and the German word 'Brot' lies the same idea 'breadness'. But this idea doesn't exist, as we shall see in the next chapter on language. We only have numerals; there is no 'number idea' behind them. But this doesn't forbid that we go from one number system to another, since we can make correspondences between them, i.e. 1=I, 2=II, 3=III, 4=IV, 5=V etc. The naïve attempt at reducing numbers may then continue asking whether this doesn't show that the *correlations* are more fundamental than the numbers. Just as mentally retarded people that work in social work places, when they have to put 5 small things into a big box, often use a piece of wood with 5 holes in it: when every hole is filled, they know they can empty it above the big box: the *correlations* between the hole and the object are more important than number itself. But this is of course nonsense, assumed is still the *number* of correlations (1=I is the first correlation, 2=II is the second correlation etc.).

The second, more serious attempt is that of logicism. Frege, Peano, Russell and Zermelo-Fraenckel all did a thing like this. Logicism said, all it needed to understand the natural numbers were the empty set and the successor function 's'. The number 0 is then identified with the empty set {ɵ}, the number 1 with the union of this empty set with itself[84] {ɵ, {ɵ}}, the number 2 as the

[83] See Spengler, *Untergang des Abendlandes*, p. 75.
[84] Set theory argues that starting with the empty set as the first element is a good application of Occam's Razor (See Van Dalen, 1977, p. xxx). To me – calling the empty 'set' an element is just empty wordplay. I

union of this set with the empty set { ɵ, {ɵ, {ɵ}}}[85]. Written differently, using the successor function ‘s’, zero is 0, one is s(0), two is s(s(0)) etc. But of course number doesn’t go away due to the operation of this function, because this function still presupposes the *number of times* it has to be applied to the number ‘0’. This function makes 2 the second number (second application of the function), three the third number (third application of the function), i.e. makes cardinal numbers ordinal numbers. But number itself is neither cardinal nor ordinal, neither spatial nor temporal. Although the number sign has an indexical spatio-temporal side, it is at the same time iconical, and its iconic order is neither cardinal nor ordinal. This iconic order is there *before* logicism formulates its set theoretic reduction of numbers.

The third attempt to reduce number to something more fundamental is that of algebraic group theory. The precise definition of a group need not worry us here. Let’s just take an example of a group. Let’s take multiplication by two and only looking at the last digit: **2**, **4**, **8**, 1**6**, 3**2**, 6**4**, 12**8**, 25**6**, 51**2** etc. We get the repeating cycle: 2, 4, 8, 6. Now take a square, divide it in 4 equal halves (for new squares) and give them all a different colour.

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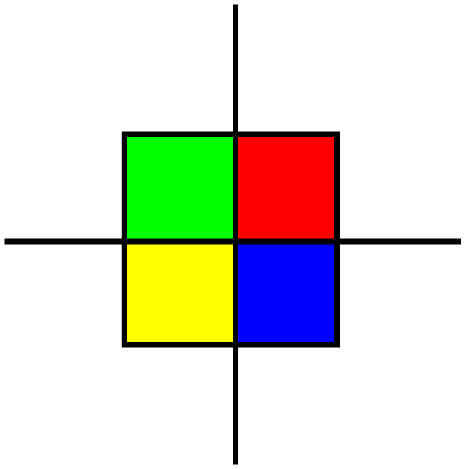
agree with Poincaré [1854-1912] ‘Later generations will regard Mengenlehre (set theory) as a disease from which one has recovered.’ See *The Mathematical Intelligencer*, vol. 13, no. 1, Winter 1991.

[85] Frege thinks of sets as characterized by a property. For example the set of *red* things is characterized in terms of the property of *redness*. What do we now mean by the actual number three? Frege explains this by the property of ‘threeness’. ‘Threeness is the property of *collections* of objects, i.e. it is a property of *sets*: as set has this particular property ‘threeness’ if and only if the set has precisely three members. The set of medal winners in a particular Olympic event has this property of ‘threeness’ for example. So does the set of tires on a tricycle, or the set of leaves on a normal clover. (...) What, then, is Frege’s definition of the actual number three? According to Frege, 3 must be a set *of* sets: the set of *all* sets with this property of ‘threeness’ (Penrose (1999), p. 131).’

With this understanding of ‘three’ Frege turns himself against the ‘Pfefferkuchen- oder Kieselsteinarithmethik (Frege (1987), p. 21)’ of both the empiricist John Stuart Mill, who identifies numbers always with numbers of something and any philosopher who understands numbers as abstractions. By doing this, however, he introduces a third realm that is just as incomprehensible: since how can we and the sets of natural objects get in touch with this realm? By participation? And are these notions of sets and threeness of sets in Frege’s setup really not abstractions?



Rotate the figure around its centre by 90 degrees. Do this 4 times, and you get the original figure back. Algebraic group theory learns that these two examples are examples of the same group: there is a correspondence between the operations: a multiplication by 2 brings the first system in an equivalent condition as a rotation by 90 degrees the second system. It is the operations that count not the type of elements. Hence numbers are just as fundamental as coloured squares. Until you see what this group is called: SO-4 (standard-orthogonal 4).[86] The 4 means the number of conditions in which the system can be due to its main operation. Hence the number pops up again after the supposed reduction. We even wrote above already: it is the operations that *count*.

Number seems to be there all along. When we try to reduce them, they suddenly pop up again. Mathematics is always already about numbers. Although numbers only exist – Darwinistically speaking – insofar as they are manipulated, at the same time they are already there when we do math. That may point towards that what Aristotle says when he calls numbers ‘archai’ (and thus not things!).[87] We translate this word – subjectively – as principle: a thing we grasp before we start doing the things we have to do. But this is definitely a wrong translation, first of all because Aristotle doesn’t know the Cartesian subject yet. Heidegger, in a gloss of Aristotle’s *Metaphysica A*, the book on ‘archai’, distinguishes 6 senses of the word ‘archai’.[88]

[86] There also exists for instance a SO2-group. Multiplication by 4 (**4**, 1**6**, 6**4**, 25**6**, 102**4**) and a rotation by 180 degrees of a square with a different top and bottom side, are examples of this.
[87] See Aristotle, *Analytica Priora* I.10.
[88] See Heidegger, GA 22 (‘Basic concepts of ancient philosophy’), p. 34.

Four of them are of importance to understand what numbers as *archai* are. At first, it looks like the first senses of this word are more *ordinary* than the later senses.

The *first* sense, Heidegger distinguishes, is 'beginning' as in the 'beginning of a road'.

In the *second* sense this (ontological) 'beginning' is narrowed down to a 'beginning' of learning, i.e. the starting point for learning (Greek *mathein*!). The starting point for learning, the starting point to become acquainted with math, is of course number.

The *third* sense is 'that with which something's genesis starts, the 'foundations' of construction work, the keel of a ship (…) and in such a way, that this 'beginning' remains within it, exists together with it.'[89] Number remains fundamental for all other branches of math: fractals, geometrical figures, probabilities are inconceivable without number; number is still present in them.

And the *final* sense 'that out of which something is perceived primarily. (...) The common, the first out of which – in a certain sense earlier than – (...) in the different orders of being, becoming, evolving and being known.' Number is neither something of the things nor something of our minds. All things can be counted, can be numbered [90] – before we even know *what*[91] they are (remember that numbers weren't abstractions of things) and all 'our' mathematical structures, fractals, geometry, probabilities presuppose number. In every attempt to get rid of number, in the end number popped up again, shining victoriously. Although our numerals only exist – Darwinistically speaking – insofar as they are used by us, we cannot reduce number at will to something else. It is there in math we learn. In that sense number rules us.[92] Thus indeed, number is not something of the things, nor something of us. Number is in-between.

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[89] Ibid.

[90] Leibniz, quoted by Frege, *The Foundations of Arithmetic*, Reclam, 1987, p. 54 'Nothing can be weighted that lacks force and parts; accordingly, nothing without parts has a measure; but there is nothing, which does not allow numbering [die Zahl zulässt]. In this way number is so to speak the metaphysical figure.' This should not be taken as if Leibniz just says we can apply number to everything, but also that only those things really are, that are numerically determinable. (Think of his numerical calculations of even 'love' or 'grace'.)

[91] There is a difference between countable nouns (things) and mass nouns (things). Examples of countable things are of course sheep, trees and books, examples of mass nouns milk, butter, water and snow. These words are treated differently *in grammar*: mass nouns often don't have a plural. But the border is fuzzy; we can speak of stone (a piece of stone) and stones (small stones). Nevertheless are mass nouns quantifiable too: we speak of more and less milk and even of *one* liter of milk etc (or even trillions of milk molecules). In this grammatical difference seems to return the differences of *speaking* of one thing as countable as in *arithmetic* and of one thing as divisible as in *geometry* – as we saw in Aristotle.

[92] Eric Temple Bell's words are thus only one-sided 'If "Number rules the universe" as Pythagoras asserted, Number is merely our delegate to the throne, for we rule Number.' In H. Eves *Mathematical Circles Revisited*, Boston: Prindle, Weber and Schmidt, 1971.

The many senses of the word ‘arché’ allow Heidegger to read the word ‘arche’ *formally*[93] as ‘the first Out-of-which and the last At-which-back.’ This formal sense is *more ordinary* than any of the other senses. Number is not only one the first thing we learn in mathematical education [‘Out-of-which], but all mathematical structures ‘depend’ ultimately on number, in all math is number [‘At-which-back]. It is in all things and in all our mathematical knowledge. Number is in-between. Number is *pre-ordained*.

This is not an arbitrary analysis of numbers. The Greek word for number ‘arithmos’ has the same root as the word ‘arché’. Both go back to the Indo-European root *ar(i), which is akin in meaning to the German verb ‘Fügen’ (Fügung = Fore-, Pre-ordaining)[94].

But how can number be an in-between? How can number be so central to math? Is mathematics something number like or is number something mathematical? Maybe, we can see this more clearly if we look at where math is an in-between, in science.[95]

§3.42 On Mathematics and Science

We already touched upon the relation that is central to this section numerous times, now after all these detours, maybe we are in a position to get a closer look at this relation.

At university math is delegated to the faculty of natural sciences. All these sciences are sciences insofar as they are mathematical; is mathematics now itself a science, i.e. a super science, or the ‘un-scientific’ basis of science, a method c.q. technique? And what is so ‘natural’ about it? Is it, as the basis of the natural sciences, even more natural – supernatural – or just a tool, an artificial construct, the Procrustean bed on which nature is forced to accommodate itself? But how is that possible that nature is porous to the needles of numbers? Or the other way round, how can these numbers needles ever pierce anything, if mathematics is just a play with identities devoid of any ‘factual content’, devoid anything in reality?[96] But if this is true, how come that the

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93 Idem, ‘*Formal* meaning of arché.’ On Heidegger’s method of the formal indication, Th. C. W. Oudemans opzoeken.

94 Private communication with Lucien ter Beek.

95 See Heidegger, *Frage nach dem Ding*, p. 54 ‘We have long since become used to think of numbers when we think of mathematics. Clearly, there is a close relationship between mathematics and numbers. The question remains only: Does this relationship exist because mathematics is something numerical, or because – on the contrary – the numeric is something mathematical? The second thing is the case.’

96 See Carl. G. Hempel, the famous philosopher of science, ‘The propositions of mathematics have, therefore, the same unquestionable certainty which is typical of such propositions as “All bachelors are unmarried,” but they also share the complete lack of empirical content which is associated with that certainty: The propositions of mathematics are devoid of all factual content; they convey no information whatever on any empirical subject matter.’ *On the Nature of Mathematical Truth* in J. R. Newman (ed.) *The World of Mathematics*, New York: Simon and Schuster, 1956. Compare Leibniz insistence of all analytical

mathematical laws of physics *do* predict natural phenomena as well as they do?[97] What *is* the relation of mathematics and science?

The first philosopher to explicitly pursue this question was Aristotle. We already quoted the classical locus where Aristotle describes this relation at length. Physical bodies are primary; the physicist studies them *insofar* they are subject to motion, the mathematician *insofar* they are unmovable. The mathematician does study the surfaces, volumes, lengths and point of physical bodies but he does not consider them *as* the surfaces etc. *of* physical bodies. But there are also certain branches of knowledge of nature that do study surfaces, lines etc., but *as* physical, such as astronomy, optics and music (the theory of harmony). This becomes evident if we look at the math used in ancient astronomy. For us the sine and cosine are just mathematical functions like all other functions, for the Greeks (and Babylonians) they are always sky angles.[98] There are only two real mathematical branches, arithmetic and geometry and these are for Aristotle of minor importance. Mathematics is just one of the many ways to study physical bodies, and because mathematics doesn't study them as *physical* bodies, one of the lesser interesting.

This outlook remained normative during the early Middle Ages, although there are some small shifts. Sciences such astronomy and optics are called *scientiae mixtae*. *Mixtae*, because they contain both elements from physics and mathematics. During the later Middle Ages and the beginning of the scientific revolution the mathematical element becomes more and more important. Roger Bacon (1214-1292) is his work on Optics, one of the *scientiae mixtae*, for instance said that 'mathematics is the door and the key to the sciences.'[99] And similar words can

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and thus mathematical proofs being identities. See also Goethe 'Mathematics has the completely false reputation of yielding infallible conclusions. Its infallibility is nothing but identity. Two times two is not four, but it is just two times two, and that is what we call four for short. But four is nothing new at all. And thus it goes on and on in its conclusions, except that in the higher formulas the identity fades out of sight,' in J. R. Newman (ed.) *The World of Mathematics*, New York: Simon and Schuster, 1956, p. 1754.

[97] See Einstein, *Geometry and Experience*, p. xx 'How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?

[98] See *Classics of Mathematics*, Ronald Calingher (ed.), Prentice Hall London 1995, p. 176. The sine and cosine only go from 0 to 180 degrees (not 360 degrees, a full circle as in our math), because the 180 degrees are sky angles.

[99] See http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Bacon.html, consulted at 2 March 2006. See also *Opus Majus* part 4 *Distinctia Prima cap 1*, 1267 'For the things of this world cannot be made known without a knowledge of mathematics.' But see also John Fauvel and Jeremy Gray (eds.) *A History of Mathematics: A Reader*, Sheridan House, 1987 'In the mathematics I can report no deficience, except that it be that men do not sufficiently understand the excellent use of the pure mathematics, in that they do remedy and cure many defects in the wit and faculties intellectual. For if the wit be too dull, they sharpen it; if too wandering, they fix it; if too inherent in the sense, they abstract it. So that as tennis is a game of no use in itself, but of great use in respect it maketh a quick eye and a body ready to put itself into all postures; so in the mathematics, that use which is collateral and intervenient is no less worthy than that which is principal and intended.' Hence, Bacon still considers mathematics to be only a tool, inferior to that what is principle and intended (physics).

be found in the work of Leonardo da Vinci[100], Kepler[101] and Galileo[102]. The famous historian of Science Alexandre Koyré[103] speaks of the influence of Platonism and its emphasis on math against Aristotelian natural science in the work of these thinkers. And indeed we see for instance Kepler – always looking for Reason, for harmony in nature – again and again trying to fit the five planets on the ratios of the five platonic solids until he found his law of the elliptic curves. Be that as it may, in the end all of them were still using math to better understand a *non-mathematized* nature. Kepler still thought that in planets and stars an intelligent agency had to be present to calculate their regular trajectories; Galileo understanding of matter is Aristotelian to and through, 'moon' matter belongs to the moon, 'Venus' matter to Venus etc., he still believes in natural places.[104] It is only with Descartes that the fundamental breach occurred that is still decisive for us.[105]

Descartes on mathematics

Descartes' philosophy is well known. By doubting about everything that is not clear and distinct he finds his *fundamentum inconsussum veritatis*, he finds his cogito. This cogito is always a 'cogito me cogitare'.[106] In everything I think of I bring myself along, in such a way that I concentrate everything I think of *back* at *myself*. And what is the cogito thinking about? We know the answer: about that which is in itself clear and distinct of the object. That is, what is

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100 Leonardo da Vinci (1452-1519) 'No human investigation can be called real science if it cannot be demonstrated mathematically.' And 'Whoever despises the high wisdom of mathematics nourishes himself on delusion and will never still the sophistic sciences whose only product is an eternal uproar.' In N. Rose *Mathematical Maxims and Minims*, Raleigh NC: Rome Press Inc., 1988.
101 Kepler (1571-1630) **'**The chief aim of all investigations of the external world should be to discover the rational order and harmony which has been imposed on it by God and which He revealed to us in the language of mathematics.' See the Mathematical quotation server, http://math.furman.edu/~mwoodard/mquot.html.
102 Galileo (1564 - 1642) '[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.' Star Messenger [*Opere Il Saggiatore*] p. 171.
103 See Alexandre Koyré, 'From the closed universe to the infinite universe,' 1957, p. xxx.
104 See my article 'Galilei Simplicius – Aristotelianism in Galileo's cosmos,' forth coming.
105 See also Heidegger, *Frage nach dem Ding*, p. 'Contrary to the attempts that pop up once in a while to let modern philosophy start with Meister Eckhart or somewhere in between him and Descartes, we should stick to the starting point just mentioned before [with the philosophy of Descartes].' I will follow Heidegger in my reading of Descartes.

Galileo's method was much better than Descartes's in his physical texts attached to the *Discours on Method* (*Optics*, *Meteores*), and much more akin to what Descartes actually *says*. Descartes own idea of matter as having itself only quantitative mathematical shapes turned out to be useless too (Newton made the big step to accept qualities in nature, but only insofar they could be understood *mathematically*. In that sense he is a true Cartesian). Nevertheless, Descartes finds *words* for what is happening in his era and as such he is the first philosopher of the modern times.
106 See for instance Heidegger*, Nietzsche II*, p. 115ff.

extensional, what is mathematically determinable about them. Mathematics always provides access[107] to nature because there is nothing that does not allow numbering. Mathematics is furthermore certain, because its theorems and axioms are evident.

However, as well-known as this exegesis of Descartes might seem, it isn't completely self-evident. The crucial step from the *cogito* to the *res extensa* and the mathematization of nature isn't a step that can be deduced logically. Furthermore, Descartes found his universal mathematics before he found the cogito: in his posthumously published *Rules for the Direction of the Mind* he already speaks about universal mathematics, but *not* about the cogito, although its presence shines already in a concealed way in this book. For instance in rule number 3 we read: 'As regards any subject we propose [objecta proposita] to investigate, we must inquire not what other people have thought, or what we ourselves conjecture, but what we can clearly and manifestly perceive by intuition or deduce with certainty. For there is no other way of acquiring knowledge.' In the elucidation of this rule it becomes clear that Descartes means *mathematical* intuition when he speaks of intuition.[108]

This rule should not be taken as if Descartes wants to apply math even more rigorously to nature than his predecessors did. No, Descartes teaches us what from now on a thing really *is*. Only what can be seen clearly, what can be seen mathematically of a thing really *is*, only what is mathematical of a thing is *true*. And what is a thing itself? A thing is an *objecta proposita*.[109] Proposed to whom? Descartes doesn't tell us in *Rules*.

We immediately think of the cogito; the objecta proposita as the new *true* object finds its foundation in the cogito as the *fundamentum inconcussum v e r i t a t i s* . However, there is no logical deduction that brings us from the *objecta proposita* to the *cogito*. Hence there is no logical deduction from the cogito to mathematics, or the other way round. In this regard Heidegger

[107] Compare Francis Bacon, *Advancement of Learning* book 2; *De Augmentis* book 3. 'And as for Mixed Mathematics, I may only make this prediction, that there cannot fail to be more kinds of them, as nature grows further disclosed.' Bacon doesn't yet see that the disclosing is itself mathematical. Compare footnote 90 on Leibniz.

[108] See Descartes, Rules for the Direction of the Mind, '*By intuition* I mean, not the wavering assurance of the senses, or the deceitful judgment of a misconstructing imagination, but a conception, formed by unclouded mental attention, so easy and distinct as to leave no room for doubt in regard to the thing we are understanding. It comes to the same thing it we say: It is an indubitable conception formed by an unclouded mental mind; one that originates solely from the light of reason, and is more certain even than deduction, because it is simpler (though, as we have previously noted, deduction, too, cannot go wrong if it is a human being that performs it). Thus, anybody can see by mental intuition that he himself exists, that he thinks, that a triangle is bounded by just three lines, and a globe by a single surface, and so on; there are far more of such truths than most people observe, because they disdain to turn their mind to such easy topics.'

[109] As regards his new use of the *word* intuition, Descartes says the following: 'Some people may perhaps be troubled by this new use of the word *intuition,* and of other words that I shall later on be obliged to shift away from their common meaning.' *Objecta* is one of the other *words* whose meaning shifts, *objecta* becomes an *objecta proposita.*

speaks of 'die nächste Nähe' of the self-evidence of mathematics and the self-evidence of the cogito.[110] Although the self-evidence of the cogito and of mathematics are absolutely different (what is more different than the knowledge of myself and the knowledge of a mathematical formula?), they somehow belong together. The one evokes the other. They belong more originally to each other than any logical reasoning can by deducing retrieve.

For Leibniz this nearest nearness, this closest closeness, has already become self-evident. Just before the formulation of his two grand principles, the principle of contradiction and the principle of sufficient reason, Leibniz says: 'It is also through the knowledge of necessary truths, and through their abstract expression, that we rise to acts of reflection, which make us think of what is called I, and observe that this or that is within us: and thus, thinking of ourselves, we think of being, of substance, of the simple and the compound [i.e. mathematics] (…). And these acts of reflection furnish the chief objects of our reasonings.'[111] Although even in his formulation it remains unclear whether the necessary truths (such as mathematics) are first and cause us to rise to knowledge of ourselves (the cogito), or the other way round that by thinking of ourselves we come to know mathematical truths.

Mathematics, reflection and replication

Mathematics and the reflection of the cogito thus form a close bond, so close that it is almost irretrievable. In Darwinism the same close bond exists between replication and mathematics.

In the previous chapter we wrote that in Darwinism the Cartesian cogito had turned flesh. Darwinian replication and Cartesian reflection are closely akin. The word 'replication' is attested 10 years earlier in the English language than reflection, and meant originally something like: 'folding back'.[112] Reflection meant something like 'bending back'.[113] We explained Darwinian replication by feed-back. The genome codes for a phenotype and this phenotype has to *answer* to the challenges of the surroundings ('has to give a "reply" to these challenges'); only those genes that code for fit enough phenotypes survive because only those phenotypes replicate themselves;

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[110] See Heidegger, *Zollinoner Seminars*, p. 138.
[111] Leibniz, *Monadology* §30. See also *Theodicy* Preface (Gerhardt vi. 27.).
[112] Replication c.1374, 'action of folding back,' also 'legal reply, rejoinder' (c.1386), from Anglo-Fr. *replicacioun,* O.Fr. *replication,* from L. *replicationem* (nom. *replicatio*) 'a reply, repetition, a folding back,' from *replicatus,* pp. of *replicare* 'to repeat, reply,' lit. 'to fold back' (see reply). Meaning 'copy, reproduction' first recorded 1692. *Replicate* 'to make a replica of' is from 1882; specifically of genetic material from 1957.
[113] Reflection, c.1384, in reference to surfaces, from L.L. *reflexionem* (nom. *reflexio*) 'a reflection,' lit. 'a bending back,' from L. *reflex-,* pp. stem of *reflectere,* from *re-* 'back' + *flectere* 'to bend.' Meaning 'remark made after turning back one's thought on some subject' is from 1659. The verb *reflect* is recorded from 1412, originally 'to turn aside;' meaning 'to turn back' an image or light rays is from 1530; sense of 'to turn one's thoughts (back) to' is first attested 1605.

the surroundings is fed back to the phenotype, and the success of the phenotype is fed back to the genes. All the things the genes fight against, are thus fed back to these genes. In the Cartesian reflection all objects the cogito thinks about are also fed back to itself, cogito *me* cogitare.

We saw that there is no mathematics without signs. Signs, however, do not arise until the first replicators, and thus mathematics too. The great mathematician and scientist Kepler still thought that in planets and stars an intelligent agency had to be present to calculate their regular trajectories. For us this is a strange anthropomorphism, the planets just move the way they do, there is no need for a spiritual agency behind them.[114] For the planets *themselves* the number of their revolutions is not a number. Only for *replicators*, like the small creatures in the desert, can the number of revolutions (for instance of the sun, i.e. the earth) become a number. That is, can the revolution be seen *as* revolution, since only by cutting the continuous movement of the planets in phases can a revolution be *a* revolution (one whole revolution), or 13 or 17 revolutions. Hence, the heat in the desert being taken as *a sign* of the number of revolutions, math arises immediately. The sign as sign brings along math.

Just as the Cartesian reflection of the cogito evoked mathematics, so Darwinian replication naturally brings along math. Or the other way round, the mathematical signs are only signs when they are taken as such by a 'reflector'. The *number* of revolutions only exists because the life and replication of the cicadas depend on it.

The example of the cicadas may seem like a lucky example to illustrate this. But does this hold true for all forms of replication? Does all (Darwinian) replication bring along number? Yes, it does. Not only because to understand the *word* replication, 'one becomes two', we need numbers, but also if we think about very primitive replicators. Think of replicating molecules that are competing. *For them* the atoms they find are not just atoms, but the *two* hard ones they need to complete their replication, for them other molecules are already those two 'friends' they need for their own replication or the 'enemies' they should avoid.[115]

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[114] Still, the idea that physical entities, for instance particles, somehow calculate is still present in fundamental physics, because the paradigm of causality is still that of contact mechanics. Particles (for instance a positive and a negative) communicate 'information' with each other about their position, charge and mass. This communication is done by (smaller) particles. These particles have to communicate their properties too; hence the catalogue of elementary particles expands and expands. A fruitful research program, with a completely unintelligible idea of information exchange behind it.

[115] Remember the RNA hyper-cycles we described in chapter 2.4, the highly interdependent replication of molecules that do not as yet have a shell around them, but whose form is thus that they can function as enzymes for each other. Cycles such as (with A, B, C, D, E, F molecules) $A + 3B + E \rightarrow A + C$, $F + C \rightarrow A + E$, $E + D \rightarrow F + 2B$ etc. Maybe it is better to say that not only each of the replicating molecules takes the other as something within the dimension of number but that the molecules themselves are as (numeral) signs to each other.

Thus, reflection (replication) opens up the sphere of number and number is (one of) the dimension of reflection. Replication it is never what it *is*, it always still has to become something (another copy etc.), and as such it is *in-between*. It opens up a space-time, between the world and another copy of itself. Number is (one of) the dimensions of this space-time. As such, number is neither of the things in the world, nor of the replicators, it is in-between too. Number is in-between because reflection is in-between and the other way round. This becomes very clear in the formal reading of number (arithmos/archai) we gave, as the first 'Out-of-which' and the last 'At-which-back'. This formal reading makes numbers almost reflexive themselves. They are only in replication, and replication through them.

But still, this doesn't answer all our questions, how can number be this in-between? And what is the relation of number to math? How can nature be porous to math? Does there remain any difference between Darwinian and human math?

Mathematics as such

Given the description of number as in-between, we might follow Heidegger in giving a formal reading of the word mathematics.

'The μαθηματα, the mathematical, is that 'of' the things that we actually already know [was wir eigentlich schon kennen]; hence, what we don't have to get out of the things first, but in a certain sense already bring along with us. Out of this we can now understand why for instance number is something mathematical. We see three chairs and say: there are three of them. What 'three' is, is something the three chairs don't tell us, and so don't three apples, three cats or any three things whatsoever. Rather, we can only count things as three things, when we already know 'three'. Thus by understanding the 'three' as such as number, we only explicitly take note of something, we in a certain sense already have.'[116] Mathematics is thus the name of that what we

[116] Heidegger, *Frage nach dem Ding*, p. 57. This is exactly the meaning of Kant's words in the Critique of Pure Reason B10 'In the earliest times to which the history of human reason extends, *mathematics*, among that wonderful people, the Greeks, had already entered upon the sure path of science. But it must not be supposed that it was as easy for mathematics as it was for logic -- in which reason has to deal with itself alone to light upon, or rather to construct for itself, that royal road. On the contrary, I believe that it long remained, especially among the Egyptians, in the groping stage, and that the transformation must have been due to a *revolution* brought about by the happy thought of a single man, the experiment which he devised marking out the path upon which the science must enter, and by following which, secure progress throughout all time and in endless expansion is infallibly secured. The history of this intellectual revolution -- far more important than the discovery of the passage round the celebrated Cape of Good Hope -- and of its fortunate author, has not been preserved. But the fact that Diogenes Laertius, in handing down an account of these matters, names the reputed author of even the least important among the geometrical demonstrations, even of those which, for ordinary consciousness, stand in need of no such proof, does at least show that the memory of the revolution, brought about by the first glimpse of this new path, must have seemed to mathematicians of such outstanding importance as to cause it to survive the tide of

already bring along in our relation with the things around us. But doesn't this make mathematics something subjective, something *human*? No because *any replicator* brings math along with it as well. But is mathematics now not something of the 'subject' replicator? Yes and no. Mathematics is another word for the in-between as in-between. Such an in-between is only there when there are replicators.

'Our expression 'the mathematic' is always double; it means on the one hand: that which can be learnt in the described way and only in this way, on the other hand: the way of learning and proceeding itself. The mathematical is that disclosedness of the things, in which we always already sojourn [darin we uns immer schon bewegen], according to which we experience them as things at all and as such. The mathematical is that initial position towards the things, in which we decide to let the things come close to us in such a way [in der wir die Dinge uns vor-nehmen auf das hin], as they are already given to us, have to be given to us and shall be given to us.'

Here we see what happened to Descartes. The *possibility* of a mathematical design of the world was already there in Aristotle's world. But only Descartes saw what was always already there, the possibility of a mathematical design of nature. He heard that an object is really an *objecta proposita*, proposed in the direction of its transparency. This guarantees the success of the mathematical design of nature. This success seems to make it superfluous to ever ask about the mathematical again.

The question why the world is porous to math is thus a non-question.[117] Math is the porosity itself. Nature can be mathematised because it is indifferent to mathematics. We are dependent on math, not nature. 'Our natural science reaches just as far as the possibility of application of mathematical methods.'[118] And mathematical methods themselves? They only exist insofar as we use them. When we use them they immediately give us a royal road, they immediately give us a way through. And the mathematician? He is constantly in the sphere of the

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oblivion. A new light flashed upon the mind of the first man (be he Thales or some other) who demonstrated the properties of the isosceles triangle. The true method, so he found, was not to inspect what he discerned either in the figure, or in the bare concept of it, and from this, as it were, to read off its properties; but to bring out what was necessarily implied in the concepts that he had himself formed *a priori*, and had put into the figure in the construction by which he presented it to himself. If he is to know anything with *a priori* certainty he must not ascribe to the figure anything save what necessarily follows from what he has himself set into it in accordance with his concept.'

[117] The question whether our ability to do math is genetical or not and questions of the evolution of this ability, should be left to the biologists and neuro-scientists. As simple as Poincaré at the beginning of the twentieth century imagines this, is probably not true '(...) by natural selection our mind has adapted itself to the conditions of the external world. It has adopted the geometry most advantageous to the species or, in other words, the most convenient. Geometry is not true, it is advantageous.' Poincaré, *Science and Method, p. xxx.* If math were just a pre-programmed structure in our brain, how come we know other geometries? And is Euclid really that 'natural'? See footnote 29.

[118] Spengler, *Untergang des Abendlandes*, p. 78.

aporia, looking for a way out. And whether he finds his way out or not, doesn't depend on him alone. Although mathematical structures only exist insofar as we use them, *they* are the ones that sometimes show us new way out (but more often not), if we sojourn with them long enough. In that sense they only exist in virtue of themselves, they are indifferent to us too.

This indifference is the logical outcome of the mathematical design of nature. It is the result of "Descartes's " mathematical design of nature. But we saw that Descartes mathematical design of nature itself was due to his hearing a few 'basic' words differently. Is the mathematical thus bound by language?

§3.5 On language and mathematics

In this chapter on mathematics, time and time again language popped up. In three different places: 1) as we talked of algorithms, 2) as we talked about mathematical structures and 3) finally as we discussed the status of math itself.

1) The algorithms: the algorithms consist of logical steps. Logic is the logic of a language. Is it? Spengler reminds us 'Logic is always a form of mathematics and the other way round'[119]. And indeed, formal logic as such seems to have nothing to do with ordinary language. Be this as it may (see §4.2 on the relation of logic and language), we saw too that human math in contrast with Darwinian algorithms could sometimes proof that there *doesn't exist an algorithm* to calculate something (the impossibility proofs). These proofs depended language.

2) The mathematical structures: we saw that Aristotle's 'mathematical structures' were depended on language: because we can *speak* of a man *as* divisible or *as* indivisible we have the two branches of math, geometry and arithmetic. Furthermore, what could be done in geometry and arithmetic was bound by the understanding of mathematical structures as *bodies* in the Greek era and as *functions* in our days.

3) The status of math: we saw that Gödel's 'meta-mathematical' proof depended critically on the selfreferentiality of *language*. Furthermore, Gödel's formalistic distinction of meaningless 'material symbols' and their meaningful interpretation, is a Cartesian (the Cartesian subject breathing meanings into the otherwise meaningless material symbols). We saw that in Descartes a crucial breach occurred, math was no longer a way to *talk* about things insofar as they were not subject to movement, but the way *par excellence* to truly deal with objects. This was due to a different understanding and use of the words 'truth' and 'object'. Finally, still up to our days

[119] Spengler, *Untergang des Abendlandes*, p. 77.

the *words* ‘natural’ and ‘artificial’ are guiding us in talking about math. We already encountered Kronecker’s words: ‘God created the integers, all else [in mathematics] is the work of men’.[120] Integers are *natural* numbers, but they needed to be *created*; at the same time our human math is creation. But how can we *create* math, is that a *natural* faculty of man? We can see this going back and forth between the words natural and creation on many pages of this chapter, as we talked of (natural) discovery or the (artificial) construction of mathematical truths, as we talked of the difference of Darwinian (natural) math and human (artificial) math, as we talked of long (unnatural) proofs and short (natural) proofs etc. This is the power of words.

Hence, is math thus bound by language? or is this just because we talked *about* math that we get this feeling. Is not a Darwinian (i.e. mathematical) reduction of language possible too? We will expose ourselves to this question in the next chapter: on Darwin and language.

[120] As recent as last year, Stephen Hawking even used these very words as the title for his volume of primary texts of ‘mathematical breakthroughs that changed history (Hawking, 2005).’